Thermal Contact Conductance of Non-Flat, Rough, Metallic Coated Metals

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Thermal contact conductance is an important consideration in such applications as nuclear reactor cooling, electronics packaging, spacecraft thermal control, and gas turbine and internal combustion engine cooling. In many instances, the highest possible thermal contact conductance is desired. For this reason, soft, high conductivity, metallic coatings are sometimes applied to contacting surfaces (often metallic) to increase thermal contact conductance. O'Callaghan et al. (1981) as well as Antonetti and Yovanovich (1985, 1988) developed theoretical models for thermal contact conductance of metallic coated metals, both of which have proven accurate for flat, rough surfaces. However, these theories often substantially overpredict the conductance of non-flat, rough, metallic coated metals. In the present investigation, a semi-empirical model for flat and non-flat, rough, uncoated metals, previously developed by Lambert and Fletcher (1996), is employed in predicting the conductance of flat and non-flat, rough, metallic coated metals. The models of Antonetti and Yovanovich (1985, 1988) and Lambert and Fletcher (1996) are compared to experimental data from a number of investigations in the literature. This entailed analyzing the results for a number of metallic coating/substrate combinations on surfaces with widely varying flatness and roughness. Both models agree well with experimental results for flat, rough, metallic coated metals. However, the semi-empirical model by Lambert and Fletcher (1996) is more conservative than the theoretical model by Antonetti and Yovanovich (1985, 1988) when compared to the majority of experimental results for non-flat, rough, metallic coated metals. [DOI: 10.1115/1.1464565]

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1 Introduction

Metallic coatings typically offer the greatest enhancement of thermal contact conductance in comparison to other classes of coatings and interstitial materials, as well as offering several other advantages. Metallic coatings do not evaporate or migrate as may greases and oils. Nor do metallic coatings leak from joints under high loading as may low melting point eutectic alloys. Thin metallic foils are tedious to insert into a joint and may wrinkle, thereby possibly even increasing contact resistance with respect to the bare junction. Elastomeric coatings, physically vapor deposited (PVD) ceramic coatings, and anodic coatings, because of their low thermal conductivity or high hardness (or both), seldom, if ever, provide the improvement in conductance attainable with metallic coatings. Hence, metallic coatings often afford the best solution.

There have been a number of experimental investigations of thermal contact conductance of metallic coated metals. Lambert and Fletcher [1] reviewed these in detail. O'Callaghan et al. [2] and Antonetti and Yovanovich [3,4] developed theoretical models, which yield nearly identical predictions and agree quite well with their experimental results.

Although these two theories accurately predict the slope of some of the experimental results in other investigations (listed as they are discussed in Section 5.0), the theories substantially overpredict the magnitude of most of those results. This disagreement is not because of any errors in the theories by O'Callaghan et al. [2] and Antonetti and Yovanovich [3,4]. Rather, it is because these theories invoke the assumption that the contacting surfaces are nearly optically flat with varying roughness. Optically flat surfaces are those with a flatness deviation, TIR (Total Included Reading), of less than 0.3 μm, or less than 0.6 μm combined for a pair of contacting surfaces. Nearly optically flat surfaces are herein defined as those with a TIR less than 1.0 μm, or less than 2.0 μm combined for both contacting surfaces. Most of the other experimental investigations reviewed by Lambert and Fletcher [1] utilized significantly non-flat specimens.

For most applications in which thermal contact conductance is a concern, the contacting surfaces, typically referred to as "engineering" surfaces, are not optically flat. Instead, they exhibit significant non-flatness, which causes macroscopic gaps and macroscopic contact resistance, just as microscopic surface features (roughness) give rise to microscopic contact resistance. The macroscopic contact resistance often predominates.

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2 Model Development

2.1 Assumptions of the Model by Lambert and Fletcher [17].

1. Contacting surfaces are circular, macroscopically spherical (see Section 2.2 for underlying rationale), and microscopically rough with a Gaussian height distribution.
2. The contact micro-hardness, H_C, is determined from Vickers micro-hardness, H_V, and σ/m (Hegazy [5]). In practice, H_C is approximated as H_V or Knoop micro-hardness, H_K.
3. Heat flows only through solid contacts, i.e., fluid gap conductance and radiative heat transfer across gaps are negligible. Such conditions exist in vacuums and if the surfaces in contact do not differ too greatly in temperature.
4. Thermal rectification is not considered. This is the phenomenon in which conductance is greater in one direction than in the other, due to either dissimilar materials or roughness.

2.2 Selection of Spherical Macroscopic Surface Profile. A universal model capable of dealing with completely arbitrary sur-

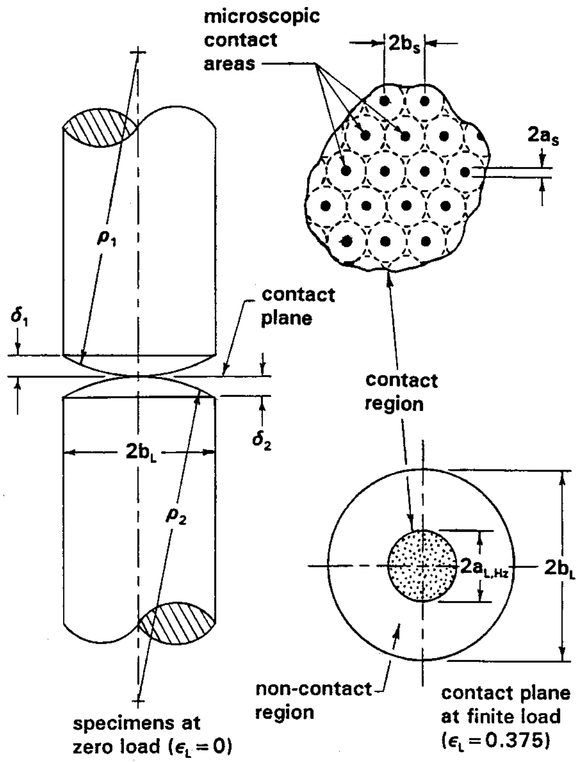

Fig. 1 Contacting spherical, rough surfaces showing the macroscopic contact radius, $a_{L,\mathrm{Hz}}$, predicted by Hertz [11], as incorporated in the models by Clausing and Chao [6] and Lambert and Fletcher [17]. Note that $a_{L,\mathrm{Hz}} \leq a_L$. Also shown is an idealized array of micro-contacts.

face profiles would probably be overly cumbersome at best, if not intractable. Wide applicability need not be sacrificed in the interest of simplifying the model. It is assumed herein that the macroscopic topography can be described by one or a few parameters, just as the microscopic topography is quite well described by combined root-mean-square roughness, σ, and combined mean absolute profile slope, m. A sphere is the simplest example, because its macroscopic profile is completely described by one parameter, its radius of curvature, ρ. This geometry is illustrated in Fig. 1.

Clausing and Chao [6], Mikic and Rohsenow [7], and Nishino et al. [8], among others, also used this simplification. This assumption is often justifiable, because nominally flat surfaces are often spherical, or at least are quite often crowned (convex) with a monotonic curvature in at least one direction.

2.3 Thermal Contact Resistance Model of Mikic [9]. Mikic [9] derived expressions for the total (microscopic, $R_{C,S}$, plus macroscopic, $R_{C,L}$) thermal contact resistance resulting from a non-uniform, axi-symmetric contact pressure distribution ($P = P(r)$), and these are given below. He did not address how to determine the pressure distribution, and this remains the crux of the problem.

$$R_{C,S} = 0.345 \frac{\sigma}{km} \left[\int_0^1 \frac{r}{b_L} \left(\frac{P}{H_C} \right)^{0.985} d\left(\frac{r}{b_L} \right) \right]^{-1} \tag{1}$$

$$R_{C,L} = 8 \frac{b_L}{k} \sum_{n=1}^{\infty} \frac{\left[\int_0^1 \frac{r}{b_L} \left(\frac{P}{P_{\mathrm{app}}} \right)^{0.985} J_0\left(\zeta_n \frac{r}{b_L} \right) d\left(\frac{r}{b_L} \right) \right]^2}{\zeta_n J_0^2(\zeta_n)} \tag{2}$$

In the present investigation, contact microhardness, H_C, and harmonic mean thermal conductivity, k, for uncoated metals are replaced by effective values, H' and k', for metallic coated metals, the latter parameters being developed by Antonetti and Yovanovich [3].

2.4 Pressure Distribution for Contact of Elastic, Rough Spheres. Greenwood and Tripp [10] developed a contact model for the elastic deformation of rough spheres. For applications where thermal contact conductance is relevant, contact loads are usually of such magnitude so as to cause only macroscopic elastic deformation. They introduced the following two dimensionless variables:

$$L^* = \frac{2L}{\sigma E' \sqrt{2\rho\sigma}} \tag{3}$$

$$P^* = \frac{P}{E' \sqrt{\sigma/8\rho}} \tag{4}$$

P^* and L^* are the dimensionless pressure and dimensionless load, respectively. The definition of P^* is not rigorously defined by Greenwood and Tripp [10] to be either the local, average, or apparent contact pressure. The definition of dimensionless load, L^*, however, is straight forward, because L is merely the contact load. The correlation developed herein employs L^*.

Hertz [11] developed a model for the pressure distribution and contact spot radius for two contacting, perfectly smooth, elastic spheres. For a given load, increasing the ratio of roughness to radius of curvature, σ/ρ, from zero (for a perfectly smooth sphere) causes an enlargement of the contact region and a reduction in the intensity of the contact pressure with respect to the Hertz [11] solution. Sasajima and Tsukada [12] defined two dimensionless ratios to characterize this behavior. $P_0/P_{0,\mathrm{Hz}}$ is the ratio of actual contact pressure (for rough spheres) to the contact pressure predicted by Hertz (for smooth spheres) at the center of contact (at $r=0$, where pressure is greatest). The ratio $a_L/a_{L,\mathrm{Hz}}$ is the actual macroscopic contact radius over the contact radius predicted by Hertz [11]. Tsukada and Anno [13] and Sasajima and Tsukada [12] give experimental and computed values of $P_0/P_{0,\mathrm{Hz}}$ for $L^* > 0.1$, that is, contacts for which sphericity is more pronounced. The model by Greenwood and Tripp [10] was used to extrapolate values of $P_0/P_{0,\mathrm{Hz}}$ for $L^* < 0.1$, as shown in Fig. 2. $P_0/P_{0,\mathrm{Hz}}$ is expressed by:

$$\frac{P_0}{P_{0,\mathrm{Hz}}} = \left(\frac{1}{1 + \frac{1}{(0.3585 \times (L^*)^{0.5840})^{1.11}}} \right)^{1/1.11} \tag{5}$$

Sasajima and Tsukada [12] developed an expression for contact pressure as a function of radial distance from the center of contact.

$$P(r) = P_0 \left[1 - \left(\frac{r}{a_L} \right)^2 \right]^{\alpha} \tag{6}$$

The exponent α is one for smooth spheres (large L^*) and is greater than one for rough spheres (small L^*) and asymptotically approaches 3. It is given by:

$$\alpha = 2 - \tanh[\log(L^*) - 1.8] \tag{7}$$

The ratio of the actual macroscopic contact radius (for rough spheres) to the Hertz [11] macroscopic contact radius (for smooth spheres), $a_L/a_{L,\mathrm{Hz}}$, is computed from $P_0/P_{0,\mathrm{Hz}}$ and α.

$$\frac{a_L}{a_{L,\mathrm{Hz}}} = \left[\frac{2(\alpha+1)}{3\left(\frac{P_0}{P_{0,\mathrm{Hz}}} \right)} \right]^{1/2} \tag{8}$$

The term b_L/a_L is the ratio of the surface radius to the actual macroscopic contact radius:

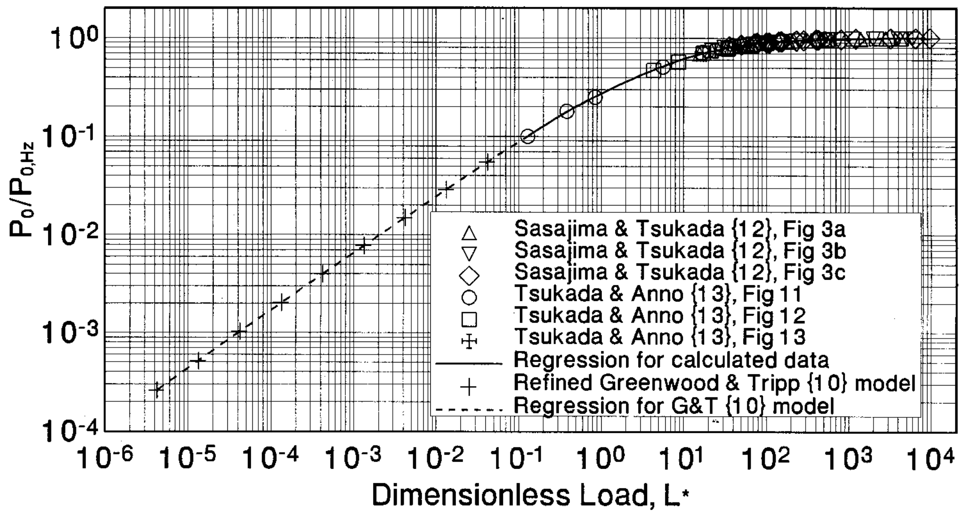

Fig. 2 Ratio of peak contact pressure $P_0/P_{0,\mathrm{Hz}}$ (at $r=0$) for rough and smooth spheres

$$\frac{b_L}{a_L}=\frac{b_L}{\left(\frac{a_L}{a_{L,\mathrm{Hz}}}\right)a_{L,\mathrm{Hz}}}=\frac{b_L}{\left(\frac{a_L}{a_{L,\mathrm{Hz}}}\right)\left(\frac{3L\rho}{4E'}\right)^{1/3}} \tag{9}$$

2.5 Model for Thermal Contact Conductance of Spherical, Rough Metals. The present model was obtained by using the contact model described above to define the pressure distribution, $P(r)$, in terms of load, mechanical properties, and surface geometry. $P(r)$ was then substituted into the expressions (Eqs. 1 and 2) by Mikic [9] for $R_{C,S}$ and $R_{C,L}$.

Buckingham Pi dimensional analysis was employed to determine the effect of each physical parameter on $R_{C,S}$ and $R_{C,L}$. This resulted in the following correlations.

$$R_{C,S}=\frac{6.15(L^*)^{0.484}\left(\frac{b_L}{a_L}\right)^2}{\left(\frac{km}{\sigma}\right)\left(\frac{L}{H_C\rho\sigma}\right)^{0.95}\left(\frac{P_0}{P_{0,\mathrm{Hz}}}\right)^{0.67}} \tag{10}$$

$$R_{C,L}=\frac{1.44(L^*)^{0.954}\left(\frac{P_0}{P_{0,\mathrm{Hz}}}\right)^{0.20}\left(\frac{b_L}{a_L}\right)^2}{\left(\frac{kL}{\rho\sigma^2E'}\right)} \tag{11}$$

Equations 10 and 11 contain $P_0/P_{0,\mathrm{Hz}}$ raised to different powers. This was required in order to linearize both $R_{C,S}$ and $R_{C,L}$ in terms of L^*, so they could be expressed as power law regressions. Thermal contact conductance, $h_{C,S+L}$, is obtained from $R_{C,S}$ and $R_{C,L}$ by:

$$h_{C,S+L}=\frac{1}{R_{C,S}+R_{C,L}} \tag{12}$$

The predictive correlations for $R_{C,S}$ and $R_{C,L}$ are applicable for any conceivable range of conditions. The dimensionless load, L^*, was varied from 4.2×10^{-5} (i.e., essentially optically flat for any realistically sized component) up to 1.3×10^4 (i.e., a smooth sphere for all practical purposes). The ratio b_L/a_L was varied from 10^{-4} up to 10^3. This covers the range of possibilities from an almost perfectly uniform pressure, where only the very center of the predicted $P(r)$ is actually brought to bear on the surface to a very small contact on a very large surface, say $a_L=1$ mm and $b_L=1.0$ m. However, for the wide range of surface measurements considered herein, which encompass nearly all likely practical situations, b_L/a_L varied between 1×10^{-1} and 2.53×10^1.

2.6 Estimation of Unspecified Parameters. Mean absolute profile slope, m, was rarely provided in experimental investigations performed in the 1960s and 1970s. To use data from those studies in the present analysis, m must be estimated. To this end Lambert [14] correlated m to σ for experiments in which both parameters were listed.

$$m_{1\text{ or }2}=0.076\sqrt{\sigma_{1\text{ or }2}\times10^6} \tag{13}$$

Uncertainty in this empirical correlation may, in extreme cases, be plus or minus a factor of two. However, profilometers capable of determining m are being used more and more in the electronics and spacecraft industries, so that estimation of m should no longer be necessary.

Also, experimental studies rarely list the radii of curvature, ρ_1 and ρ_2, of the specimen surfaces. To circumvent this difficulty, the combined radius of curvature, ρ, may be estimated from the combined non-flatness, TIR, of both surfaces. In the present study, the combined crown drop, δ, is assumed to equal TIR. See Fig. 1. Thus, ρ is:

$$\rho=b_L^2/2\delta \tag{14}$$

The concept of radius of curvature loses relevance if the surfaces are decidedly non-spherical. If this is so, the present model may substantially disagree (typically in an overly conservative fashion) with experimental data, but usually by no more than a factor of three.

For non-circular specimens, an effective macroscopic contact radius b_L' is defined as:

$$b_L'=\sqrt{A_{\mathrm{app}}/\pi} \tag{15}$$

This expression is useful for commonly utilized square or rectangular surfaces (provided the length is not, say, more than twice the width for rectangular surfaces) or less frequently encountered triangular surfaces (approximately equilateral). This method of estimating b_L for non-circular contact surfaces is supported by the work of Yovanovich et al. [15].

3 Results and Discussion

Experimental results for thermal contact conductance of silver-coated nickel by Antonetti [16] are compared to the semi-empirical model of Lambert and Fletcher [17] and the theoretical model of Antonetti and Yovanovich [3,4] in Fig. 3. Antonetti and Yovanovich's [3,4] model very accurately predicts Antonetti's [16] results because he employed nearly optically flat specimens (TIR$=1\ \mu$m). Lambert and Fletcher's [17] model reduces to An-

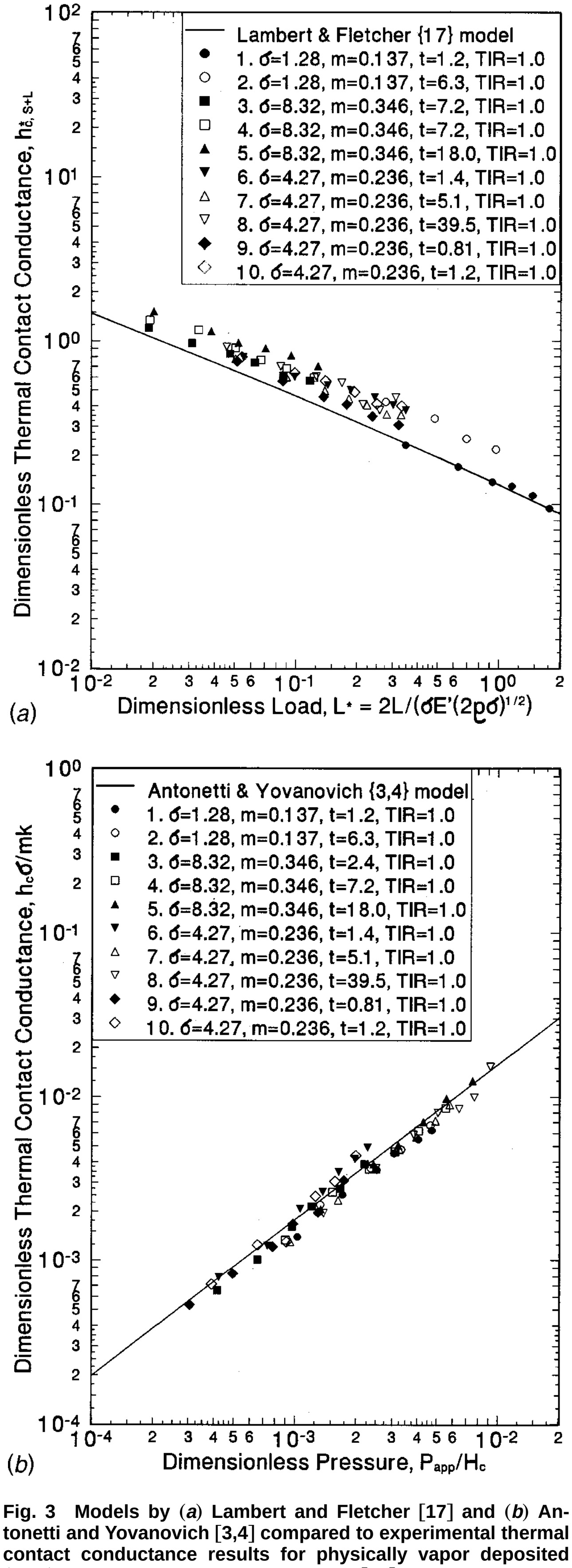

Fig. 3 Models by (*a*) Lambert and Fletcher [17] and (*b*) Antonetti and Yovanovich [3,4] compared to experimental thermal contact conductance results for physically vapor deposited (PVD) silver on nickel 200 by Antonetti [16]

tonetti and Yovanovich's [3,4] model for perfectly flat surfaces. Lambert and Fletcher's [17] model is slightly conservative (i.e., under-predictive) because it accounts for a minimal macroscopic contact resistance due to the slight non-flatness of Antonetti's [16] specimens.

Results from three investigations of metallic coated metals, Fried [18], Mal'kov and Dobashin [19] and O'Callaghan et al. [2] are plotted in Fig. 4. O'Callaghan et al.'s [2] experimental results agree well with both models, again because their data were obtained for nearly optically flat specimens for which both models are adequate. Antonetti and Yovanovich's [3,4] model over-predicts all of Fried's [18] and about half of Mal'kov and Dobashin's [19] experimental results. Lambert and Fletcher's [17] model under-predicts Mal'kov and Dobashin's [19] results and over-predicts Fried's [18] data. The significant scatter in Fried's [18] results suggests the presence of rather large uncertainties. Mal'kov and Dobashin [19] listed wide ranges of flatness deviation, TIR. Their experimental facility and data analysis are not well described, so it is not possible to estimate the accuracy of their work.

Figure 5 shows that Antonetti and Yovanovich's [3,4] model over-predicts all of Kang et al.'s [20] results, though their model accurately predicts the slope of the experiments. Lambert and Fletcher's [17] model roughly follows the mean of the data, though it does not reduce the scatter with respect to nor predict the slope as well as Antonetti and Yovanovich's [3,4] model. Kang et al. [20] employed aluminum alloy specimens with turned contact surfaces produced on a lathe. This preparation method yielded wavy surfaces that were not monotonically curved as is assumed in the model by Lambert and Fletcher [17]. Kang et al. [20] reported the typical trough to crest height of the wavy surfaces to be 10 μm, though they did not report the flatness deviation, TIR. In the present analysis TIR was assumed to equal the waviness height of 10 μm. The fact that the theory by Antonetti and Yovanovich [3,4] accurately predicts the slope of Kang et al.'s [20] experiments suggests that whatever non-flatness the surfaces exhibited was considerably smaller than the waviness. The fact that Lambert and Fletcher's [17] model predicts the mean magnitude of the data demonstrates that waviness was predominant over roughness in determining contact conductance. Kang et al. [20] noted that conductance decreased drastically as coating thickness, t, was increased from 0.25 μm to 5.0 μm, which they attributed to increased bulk resistance of the thicker coatings. However, it is much more likely that oxidation of the coating surfaces between steps during deposition of the multi-layer thicker coatings substantially increased the resistance.

As illustrated in Fig. 6, Antonetti and Yovanovich's [3,4] model substantially over-predicts Chung et al.'s [21] results by a factor of 5 to 100 and inaccurately predicts the slope, while Lambert and Fletcher's [17] model only moderately over-predicts the magnitude of the data and accurately predicts the slope. Chung et al. [21] did not report flatness deviation, TIR. The TIR value (10 μm) listed in Fig. 6 was assumed for the present study to facilitate analyzing the results by Chung et al. [21], because TIR=10 μm is typically achieved by grinding and polishing. However, it is quite possible that TIR for the specimens used by Chung et al. [21] was much greater than 10μm. They used specimens of the same aluminum alloy and size employed by Kang et al. [20], although Chung et al. [21] ground and polished their specimens instead of turning them. Grinding, lapping, and polishing operations often produce spherical surfaces. This may be why Lambert and Fletcher's [17] model accurately predicts the slope of Chung et al.'s [21] results, whereas Antonetti and Yovanovich's [3,4] theory does not.

Figure 7 shows that Antonetti and Yovanovich's [3,4] model over-predicts the experimental data reported by Sheffield et al. [22], while Lambert and Fletcher's [17] model predicts the mean and slope of the data quite well with relatively little scatter. Sheffield et al. [22] used specimens made of the same aluminum alloy and prepared in the same way as those used by Chung et al. [21].

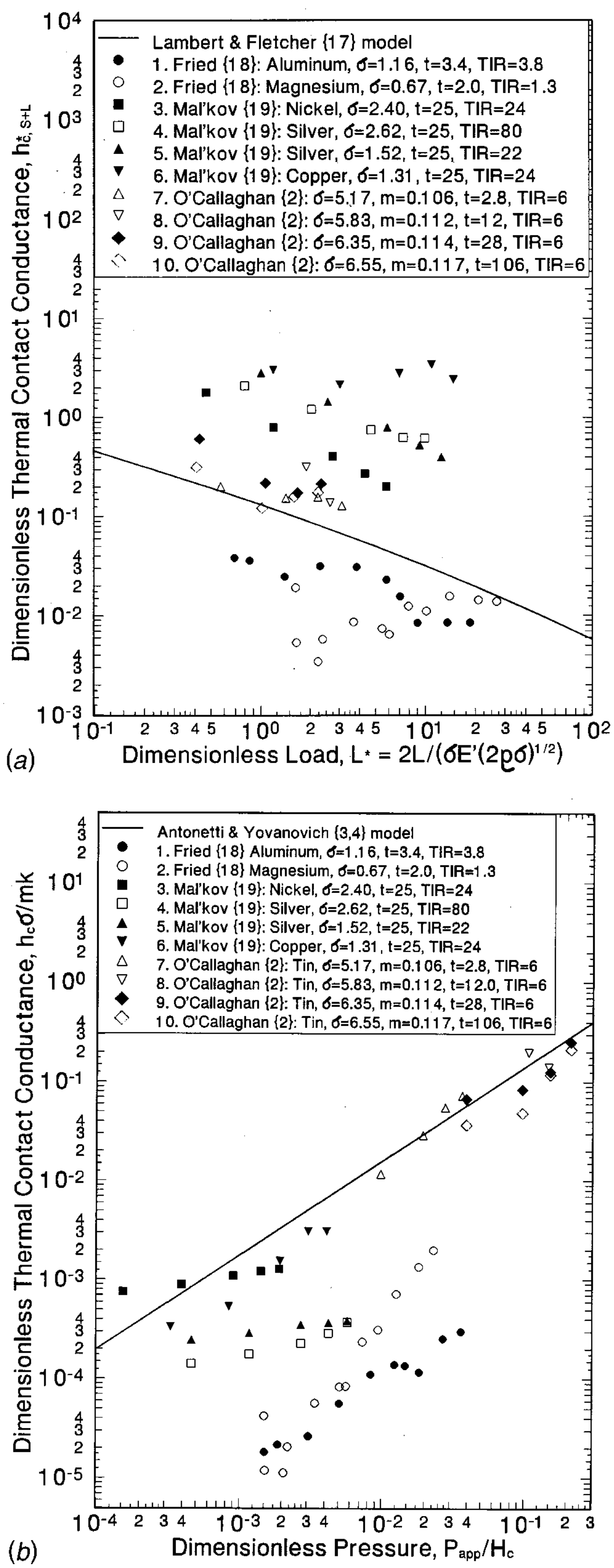

Fig. 4 Models by (*a*) Lambert and Fletcher [17] and (*b*) Antonetti and Yovanovich [3,4] compared to experimental thermal contact conductance results for PVD aluminum and magnesium on stainless steel 304 by Fried [18]; nickel, silver, and copper platings on stainless steel by Mal'kov and Dobashin [19]; and PVD tin on low alloy steel by O'Callaghan et al. [2]

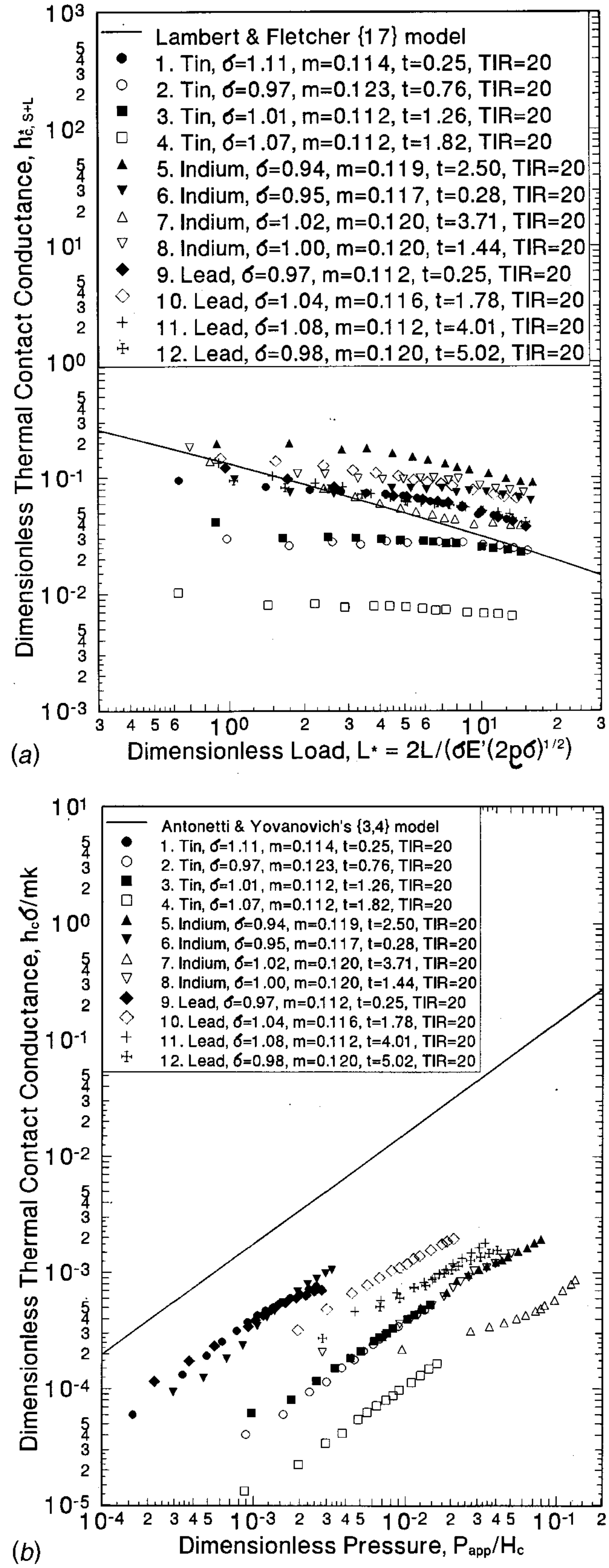

Fig. 5 Models by (*a*) Lambert and Fletcher [17] and (*b*) Antonetti and Yovanovich [3,4] compared to experimental thermal contact conductance results for physically vapor deposited (PVD) lead, tin, and indium on aluminum alloy 6061-T6 by Kang et al. [20]

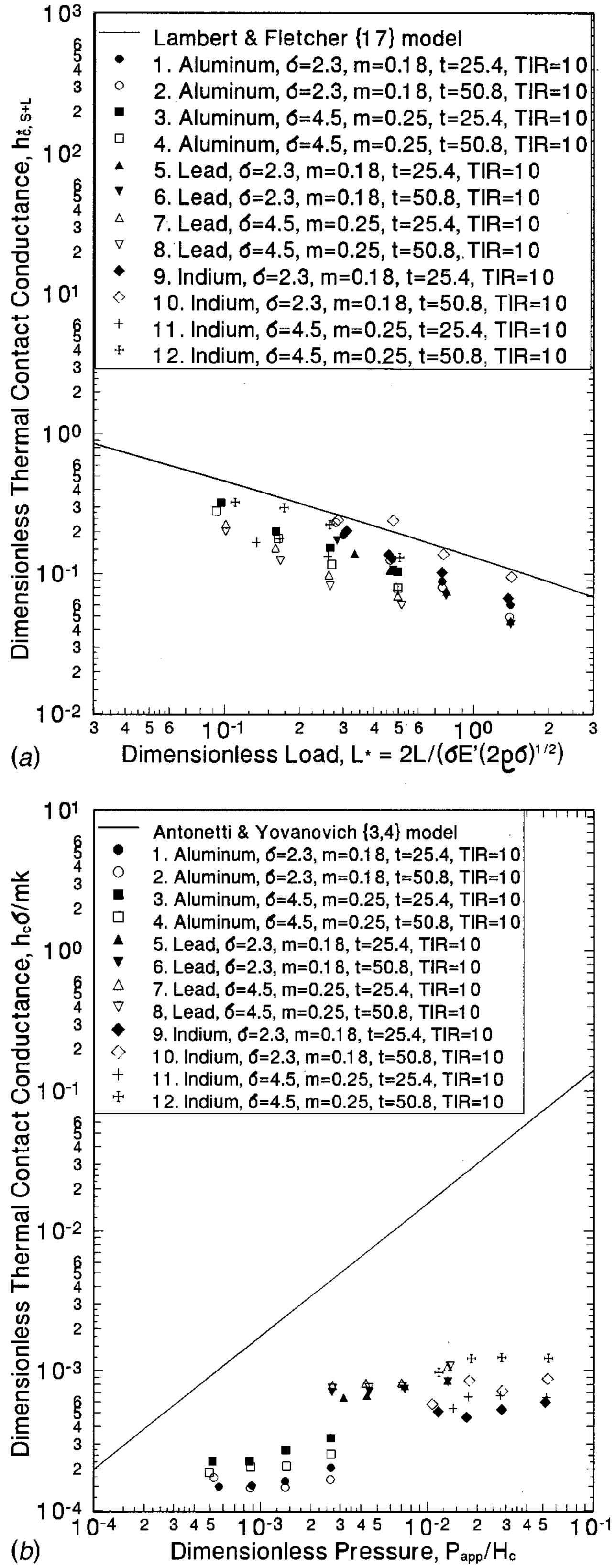

Fig. 6 Models by (*a*) Lambert and Fletcher [17] and (*b*) Antonetti and Yovanovich [3,4] compared to experimental thermal contact conductance results for physically vapor deposited (PVD) aluminum, lead, and indium on aluminum alloy 6061-T651 by Chung et al. [21]

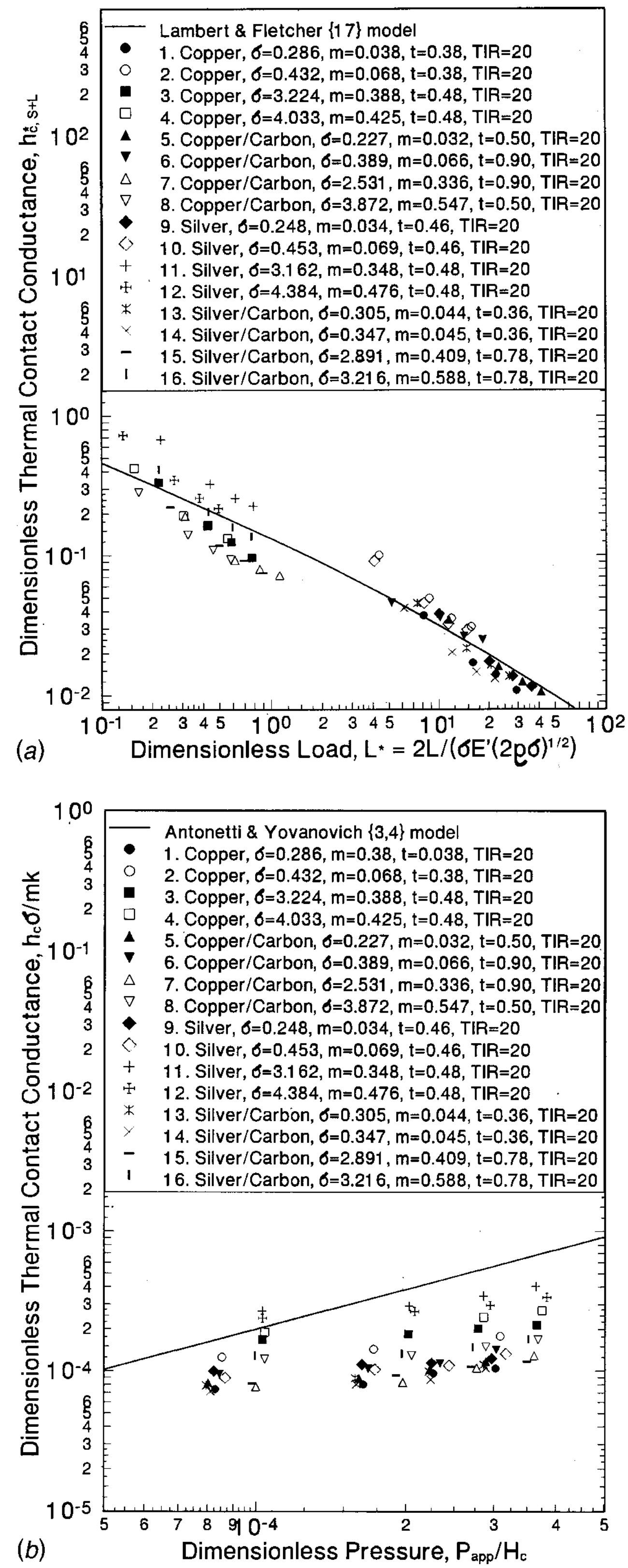

Fig. 7 Models by (*a*) Lambert and Fletcher [17] and (*b*) Antonetti and Yovanovich [3,4] compared to experimental thermal contact conductance results for (PVD) copper, copper/carbon, silver, and silver/carbon on aluminum alloy 6061-T651 by Sheffield et al. [22]

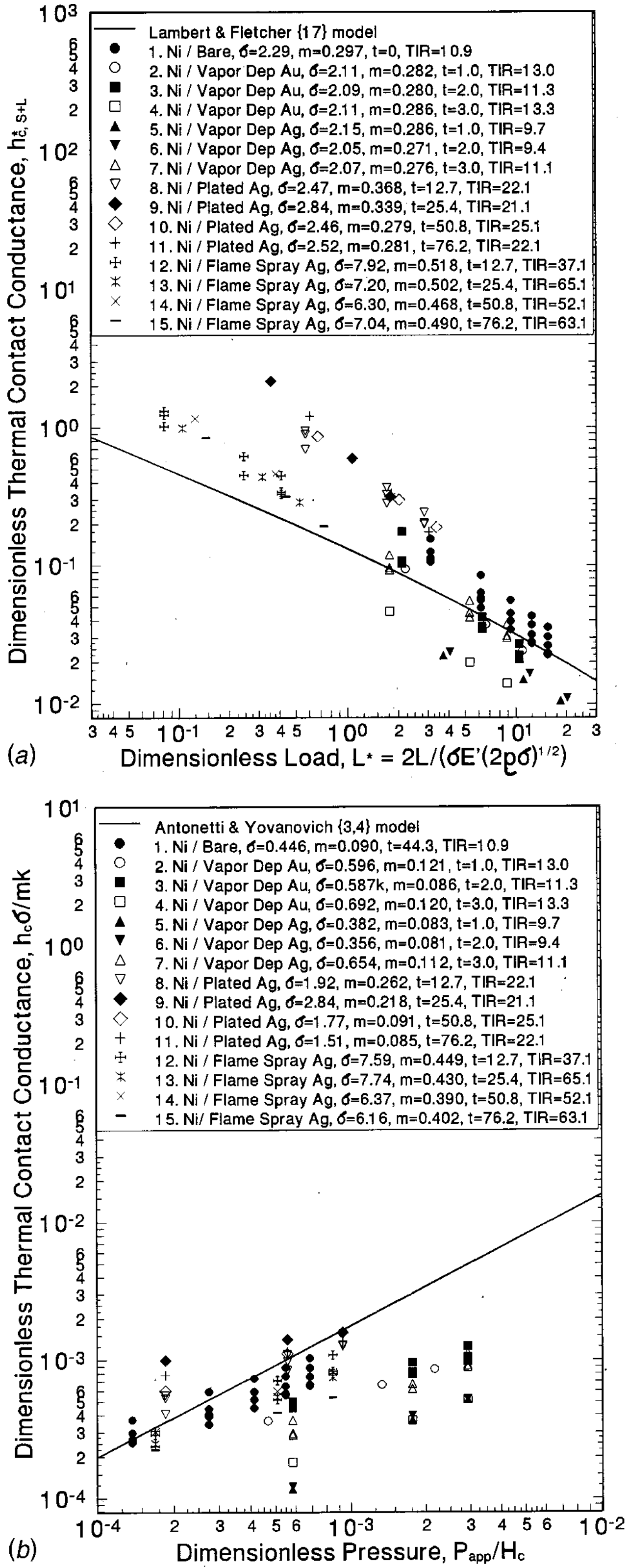

Fig. 8 Models by (*a*) Lambert and Fletcher [17] and (*b*) Antonetti and Yovanovich [3,4] compared to experimental thermal contact conductance results for electroless nickel plated copper to gold and silver coated (physically vapor deposited (PVD), electroplated, and flame sprayed) aluminum alloy A356-T61 by Lambert and Fletcher [23]

Thus, arguments similar to those made in conjunction with the results in Fig. 6 may be applied to the results in Fig. 7.

Figure 8 demonstrates that Lambert and Fletcher's [17] model is conservative (under-predictive) compared to most of the experimental results by Lambert and Fletcher [23], but their model predicts the slope of the results quite well. Antonetti and Yovanovich's [3,4] model is non-conservative (over-predictive) compared to most of the data and inaccurately predicts the slope. Lambert and Fletcher [23] prepared their aluminum alloy specimens using methods similar to those of Chung et al. [21] and Sheffield et al. [22]. Again, the arguments put forth to explain Fig. 6 also apply to Fig. 8.

Conclusions

Both the semi-empirical model by Lambert and Fletcher [17] and the theoretical model by Antonetti and Yovanovich [3,4] accurately predict thermal contact conductance of nearly optically flat (TIR$\leq 2\ \mu$m), rough, metallic coated metals. Additionally, through comparison with a large number of experimental results from the literature, the model by Lambert and Fletcher [17] is shown to provide usually conservative predictions of conductance for non-flat, rough, metallic coated metals.

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Nomenclature

a_L = macro-contact radius for rough spheres (m)
$a_{L,\mathrm{Hz}}$ = Hertz macro-contact radius for smooth spheres (m)
a_S = radius of micro-contact (m)
A_{app} = apparent contact area (m^2)
b_L = radius of surfaces in contact (m)
b_S = radius of heat flux channel for micro-contact (m)
E = modulus of elasticity (N/m^2)
E' = effective elastic modulus (N/m^2), $E' = [(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2]^{-1}$
$h_{c,s+L}$ = thermal contact conductance (W/m^{2}K), $h_{c,S+L} = 1/(R_{c,S}+R_{c,L})$
H_C = contact micro-hardness (N/m^2)
H_K = Knoop micro-hardness (N/m^2)
H_V = Vickers micro-hardness (N/m^2)
J_0, J_1 = Bessel functions of the first kind
k = harmonic mean thermal conductivity (W/m-K), $k = 2k_1k_2/(k_1+k_2)$
L = contact load (N)
L^* = dimensionless contact load, $L^* = 2L/[\sigma E'(2\rho\sigma)^{1/2}]$
m = combined mean absolute profile slope (m/m), $m = (m_1^2 + m_2^2)^{1/2}$
P = contact pressure (N/m^2)
P^* = dimensionless contact pressure, $P^* = P/[E'(\sigma/8\rho)^{1/2}]$
P_{app} = apparent contact pressure (N/m^2)
P_0 = maximum contact pressure (at $r=0$) for rough spheres (N/m^2)
$P_{0,\mathrm{Hz}}$ = Hertz' maximum contact pressure (at $r=0$) for smooth spheres (N/m^2)
r = distance from center of axi-symmetric contact (m)
$R_{C,L}$ = large scale thermal contact resistance (m^{2}K/W)
$R_{C,S}$ = small scale thermal contact resistance (m^{2}K/W)
TIR = non-flatness (Total Included Reading), (m), $\mathrm{TIR} = \mathrm{TIR}_1 + \mathrm{TIR}_2$
α = load redistribution parameter
δ = combined crown drop of surfaces (m), $\delta = \delta_1 + \delta_2$

ε_L = ratio of macroscopic contact radius to surface radius, $a_{L,\mathrm{Hz}}/b_L$
ζ_n = nth root of Bessel function $J_1(\zeta_n)$
μ = micro$=10^{-6}$, combined with meters or inches
ν = Poisson ratio
ρ = combined radius of curvature (m), $\rho=(1/\rho_1+1/\rho_2)^{-1}$
σ = combined root-mean-square (rms) roughness (m), $\sigma=(\sigma_1{}^2+\sigma_2{}^2)^{1/2}$

Subscripts

Hz = Hertz' theory for contacting smooth spheres
L = large scale, macroscopic
n = index of summation
S = small scale, microscopic
0 = at axis or center of contact
1 = specimen or surface 1
2 = specimen or surface 2

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