Uncertainty assessment of Guarded Hot Plate Apparatus LOG 6.1.08

Dr. Hans Simmler (hans.simmler@empa.ch)

Empa Building Technologies Lab, Ueberlandstrasse 129, CH-8600 Duebendorf

Introduction

The ISO Guide to the Expression of Uncertainty in Measurement (GUM) provides general methods for the assessment of uncertainties [1]. In the following sections, the GUM procedure is applied in a descriptive way to the symmetric Guarded Hot Plate (ISO 8302 [2], EN 1946-2 [3]) at Empa registered under LOG No. 6.1.08. Assuming independently measurable quantities x_i (or neglecting covariance) the combined standard uncertainty is defined as the square root of the combined variance of the evaluated quantity $y(x_i)$

$$u_c^2 = \sum_i \left(\frac{\partial y}{\partial x_i} \right)^2 u^2(x_i) \tag{1}$$

In this statistical error propagation expression the variances $u^2(x_i)$ of input quantities are weighted by the square partial derivatives, also called sensitivity coefficients c_i^2. The GUM describes two methods for the determination of the variances $u^2(x_i)$. The method type A is an empirical evaluation of the mean value x_i and variance $u^2(x_i)$ according to standard definitions. The method type B means a non-statistical approach and may be based on manufacturer information, calibration certificates, theoretical models etc.

Model

For a symmetric Guarded Hot Plate apparatus with two specimens the basic model for the evaluation of the "thermal conductivity" (actually the thermal transfer factor) is

$$\lambda = \frac{P \cdot d}{2A \cdot \Delta T} \tag{2}$$

where P heating power in the metering zone
d thickness of the specimen
A area of the metering zone
ΔT temperature difference between hot and cold surface

All quantities should be read as values averaged in time and space, sampled under quasi stationary conditions.

To account for major effects apparent in the measurement the basic model has to be extended in several respects. The heating power P may be written as

$$P = P_{spe} + P_{imb} + P_{edge} \tag{3}$$

where P_{spe} heat flow through the specimens
P_{imb} lateral heat flow due to imbalance (metering – guard zone)
P_{edge} heat loss (gain) through the specimen edges

The thickness d may be modelled as

$$d = d_r \left(1 + \alpha \Delta T_{mr}\right) \tag{4}$$

where d_r thickness at reference temperature
α thermal expansion coefficient (material or spacer), $\leq 10^{-4}$ K^{-1}
ΔT_{mr} difference between mean and reference temperature

The square metering area A is approximated by

$$A = \ell_r^2 \left(1 + \alpha_{Al} \Delta T_{hr}\right)^2 \tag{5}$$

with ℓ_r length at reference temperature
α_{Al} thermal expansion coefficient (Aluminium), $\cong 20 \times 10^{-6}$ K^{-1}
ΔT_{hr} difference between hot plate and reference temperature

For the temperature difference we assume a contribution from a surface resistance, since the contact between specimen and plate surface (containing imbedded thermocouple sensors) is not ideal:

$$\Delta T = \Delta T_{spe} - \Delta T_{sur} \tag{6}$$

with ΔT_{spe} temperature difference between the specimen surfaces
ΔT_{sur} temperature difference due to surface resistance

Thus an extended (but still simplified) model of the thermal conductivity evaluation is

$$\lambda = \frac{\left(P_{spe} + P_{imb} + P_{edge}\right) d_r \left(1 + \alpha \Delta T_{mr}\right)}{2 \ell_r^2 \left(1 + \alpha_{Al} \Delta T_{hr}\right)^2 \left(\Delta T_{spe} - \Delta T_{sur}\right)} \tag{7}$$

Note: Since the additional terms in equation (7) are not determined in detail, the resulting thermal conductivity is calculated according to equation (2). However, equation (7) or equations (3) to (6) respectively may be used to calculate a realistic range for the measurement uncertainty due to the additional effects neglected in the evaluation.

Combined standard uncertainty

Applying equation (1) to the basic equation (2), the combined variance is expressed as

$$u_c^2(\lambda) = c_P^2 u^2(P) + c_d^2 u^2(d) + c_A^2 u^2(A) + c_{\Delta T}^2 u^2(\Delta T) \tag{8}$$

with

$$c_p = \lambda/P,\ c_d = \lambda/d,\ c_A = -\lambda/A,\ c_{\Delta T} = -\lambda/\Delta T \tag{9}$$

Using equations (3) to (6), the variances in equation (8) are:

$$u^2(P) = u^2(P_{spe}) + u^2(P_{imb}) + u^2(P_{edge}) \tag{10}$$

$$u^2(d) = \left(\frac{\partial d}{\partial d_r}\right)^2 u^2(d_r) + \left(\frac{\partial d}{\partial \Delta T_{mr}}\right)^2 u^2(\Delta T_{mr}) \cong u^2(d_r) + \left(d_r \alpha\right)^2 u^2(\Delta T_{mr}) \tag{11}$$

$$u^2(A) = \left(\frac{\partial A}{\partial \ell_r}\right)^2 u^2(\ell_r) + \left(\frac{\partial A}{\partial \Delta T_{hr}}\right)^2 u^2(\Delta T_{hr}) \cong \left(2\ell_r\right)^2 u^2(\ell_r) + \left(2A\alpha_{Al}\right)^2 u^2(\Delta T_{hr}) \tag{12}$$

$$u^2(\Delta T) = u^2(\Delta T_{spe}) + u^2(\Delta T_{sur}) \tag{13}$$

According to equations (8) to (13) the combined variance may also be written

$$u_c^2(\lambda) = c_{P_{spe}}^2 u^2(P_{spe}) + c_{P_{imb}}^2 u^2(P_{imb}) + c_{P_{edge}}^2 u^2(P_{edge}) + c_{d_r}^2 u^2(d_r) + c_{\Delta T_{mr}}^2 u^2(\Delta T_{mr}) + c_{\ell_r}^2 u^2(\ell_r) + c_{\Delta T_{hr}}^2 u^2(\Delta T_{hr}) + c_{\Delta T_{spe}}^2 u^2(\Delta T_{spe}) + c_{\Delta T_{sur}}^2 u^2(\Delta T_{sur}) \quad (14)$$

with the direct sensitivity coefficients

$$\begin{aligned} c_{p_{spe}} &= c_{p_{imb}} = c_{p_{edge}} = c_p \\ c_{d_r} &\cong c_d \\ c_{\Delta T_{mr}} &\cong c_d(d_r\alpha) \\ c_{\ell_r} &\cong c_A(2\ell_r) \\ c_{\Delta T_{hr}} &\cong c_A(2A\alpha_{Al}) \\ c_{\Delta T_{spe}} &= c_{\Delta T_{sur}} = c_{\Delta T} \end{aligned} \quad (15)$$

Uncertainty budget

The variances in equations (10) to (13) are quantified and commented in table 1. If not stated otherwise, the variance was determined statistically (giving a normal probability distribution) by a series of measurements (type A). More details can be found in ref. [4].

Table 1: Standard uncertainty of quantities used in the evaluation of the thermal conductivity.

quantity	expression	remarks
$u(P_{spe})$	$\cong P_{spe} \times 0.05\% / \sqrt{3}$	calibrated power analyser (type B), rectangular prob. distribution
$u(P_{imb})$	$\cong (\Phi_0 + \lambda c) u(\Delta T_{gap})$	c.f. ref. [3]. Φ_0 = 0.534 W/K c < 3.68 m $u(\Delta T_{gap})$ < 0.001 K
$u(P_{edge})$	$\cong P_{spe} \times 0.05\%$	theoretical analysis for d < 160 mm (type B) and 0.45 < e < 0.55 (experiment.)
$u(d_r)$	$\cong 0.2\,mm / \sqrt{3}$	rectangular prob. distribution
$u(\Delta T_{mr})$	$\cong 10$ K	thermal expansion not corrected in the evaluation
$u(\ell_r)$	$\cong 0.2\,mm / \sqrt{3}$	rectangular prob. distribution
$u(\Delta T_{hr})$	$\cong 15$ K	thermal expansion not corrected in the evaluation
$u(\Delta T_{spe})$	$\cong \sqrt{2} \times 0.04\,K$	difference of absolute surface temperatures
$u(\Delta T_{sur})$	$\cong 0.0054 \times (\lambda/d) \times \Delta T$	experimentally determined relation for ΔT_{sur} [5], not corrected in the evaluation

In table 2 the sensitivity coefficients in equation (9), the standard uncertainties in table 1 and the resulting combined standard uncertainty $u_c(\lambda)$ are summarised for the reference material IRMM 440 and for a second material with a thermal resistance at the minimum specified for this apparatus ($R = 0.3\ m^2K/W$).

Table 2: Uncertainty budget for the thermal conductivity determination (examples).

Quantity	Unit	Material 1 (IRMM 440)	Material 2 (low R)
λ	$Wm^{-1}K^{-1}$	0.03048	0.200
d_r	m	0.030	0.060
α	K^{-1}	1.00E-04	1.00E-04
α_{AL}	K^{-1}	2.00E-05	2.00E-05
ΔT	K	10.0	10.0
ℓ_r	m	0.30	0.30
A	m^2	0.09	0.09
P_{spe}	W	1.83	6.00
c_p	$m^{-1}K^{-1}$	0.017	0.033
c_d	$Wm^{-2}K^{-1}$	1.016	3.333
c_A	$Wm^{-3}K^{-1}$	-0.339	-2.222
$c_{\Delta T}$	$Wm^{-1}K^{-2}$	-0.003	-0.020
$u(P_{spe})$	W	5.28E-04	1.73E-03
$u(P_{imb})$	W	6.46E-04	1.27E-03
$u(P_{edge})$	W	9.14E-04	3.00E-03
$u(d_r)$	m	1.15E-04	1.15E-04
$u(\Delta T_{mr})$	K	1.00E+01	1.00E+01
$u(\ell_r)$	m	1.15E-04	1.15E-04
$u(\Delta T_{hr})$	K	1.50E+01	1.50E+01
$u(\Delta T_{spe})$	K	5.66E-02	5.66E-02
$u(\Delta T_{sur})$	K	5.49E-02	1.80E-01
$u(P)$	W	1.24E-03	3.69E-03
$u(d)$	m	1.15E-04	1.15E-04
$u(A)$	m^2	8.78E-05	8.78E-05
$u(\Delta T)$	K	7.88E-02	1.89E-01
$c_p\ u(P)$	$Wm^{-1}K^{-1}$	2.06E-05	1.23E-04
$c_d\ u(d)$	$Wm^{-1}K^{-1}$	1.17E-04	3.85E-04
$\|c_A\|\ u(A)$	$Wm^{-1}K^{-1}$	2.97E-05	1.95E-04
$\|c_{\Delta T}\|\ u(\Delta T)$	$Wm^{-1}K^{-1}$	2.40E-04	3.77E-03
$u_c(\lambda)$	$Wm^{-1}K^{-1}$	0.27E-03	3.80E-03
$u_c(\lambda) / \lambda$	-	0.89%	1.90%

Conclusions

As can be seen in table 2 the overall uncertainty budget is dominated by the measurement uncertainty of the thickness and the temperature difference between the specimen surfaces. A particular issue with this equipment is the surface resistance that is rather large because the temperature sensors are embedded in the hot and cold plates. A further contribution comes from a Teflon protection film glued on the surfaces of the hot plate (c.f. table 1). If the surface resistance is taken into account, an (uncorrected) measurement result $\lambda = 0.03031\ Wm^{-1}K^{-1}$ for the reference material IRMM 440 at 10 °C mean temperature becomes $\lambda = 0.03049\ Wm^{-1}K^{-1}$, where the certified value is $\lambda = 0.03048\ Wm^{-1}K^{-1}$. A more serious problem occurs for low resistance specimens. This shows up in table 2 (Material 2) as a clearly increased total uncertainty, which is in fact the systematic error by the surface resistance limiting the low resistance range. Therefore a systematic correction at least for low resistance specimens will be considered.

References

[1] Guide to the Expression of Uncertainty in Measurement (GUM), ISO, Geneva 1995, ISBN 92-67-10188-9.

[2] ISO 8302 (1991): Thermal insulation - Determination of steady-state thermal resistance and related properties - Guarded hot plate apparatus.

[3] EN 1946-2 (1999): Thermal performance of building products and components - Specific criteria for the assessment of laboratories measuring heat transfer properties - Part 2: Measurements by the guarded hot plate method.

[4] B. Binder, H. Simmler, R. Vonbank: Assessment of EMPA Guarded Hot Plate apparatus LOG 1.6.08 (1997).

[5] c.f. LOG 6.1.08 / corresponding documents.