A procedure to correct the error in the structure function based thermal measuring methods

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Abstract

In this paper a methodology is presented to correct the systematic error of structure function based thermal material parameter measuring methods. This error stems from the fact that it is practically impossible to avoid parallel heat-flow paths in case of forced one-dimensional heat conduction.

With the presented method we show how to subtract the effect of the parallel heat-flow paths from the measured structure function. With this correction methodology the systematic error of structure function based thermal material parameter measuring methods can be practically eliminated.

Application examples demonstrate the accuracy increase obtained with the use of the method.

Keywords

Thermal transient measurements, RC thermal models, structure functions, thermal conductivity measurement, interface thermal resistance measurement

Nomenclature

C_Σ	[Ws/K]	cumulative thermal capacitance along a heat-flow path
d		denominator coefficients
G	[W/K]	thermal conductance
n		numerator coefficients
R	[K/W]	thermal resistance
R_Σ	[K/W]	cumulative thermal resistance along a heat-flow path
s	[1/s]	$j\omega$: complex frequency
τ	[s]	time-constant
z	[-]	logarithmic time, $t = \exp(z)$
Z	[K/W]	thermal impedance

1. Introduction

The structure function based evaluation of the thermal transient response functions 1 opened new avenues in the thermal transient testing of microelectronics structures. With the help of the structure functions die attach failures of packages can be determined [2], and they can be used to determine partial thermal resistances in a heat-flow path [3]. The structure functions can be obtained by direct mathematical transformations from the measured or simulated thermal transient response functions of the system. For a more detailed introduction to structure functions see [8] in this volume.

Several methods have been developed to measure thermal material parameters, based on the structure function evaluation [4]. In these measurements it is exploited that the values of the structure functions and their slopes depend, among others, on material parameters. If we know the geometric parameters we can determine from the structure functions the material parameters from fast and simple thermal transient measurements.

The structure functions are one-dimensional representations of the heat-flow path. One dimensional heat-flow can be forced in most of the practical cases in order to facilitate the structure function evaluation, but there is always a certain error in these measurements. This stems from the fact that there is always a certain amount of parasitic heat-flow, that is moving in other directions than the considered one dimension.

In this paper we present a procedure that can be used to consider the parasitic heat-flow that is always present in the case of structure function evaluation based measurement methods, and enables correcting the measured results.

2. A procedure to correct the regular error of structure function based thermal measurement evaluation

2.1. The source of the regular error

The common feature of the structure function based material parameter measuring methods is that one dimensional heat-flow is forced in the examined structure, by the application of appropriate boundary conditions. Heat is switched on at the $t=0$ time instant at a spot in the structure, and from that on the temperature of the same spot is recorded as the function of time, until steady state is reached. From the measured transient curves the *structure functions* are determined by direct mathematical transformations.[5]

As it was mentioned in the introduction, besides the main heat-flow path, where we force the one dimensional heat-flow, there are always more or less important *parallel heat-flow paths* where a part of the heat is lost. This lost heat is responsible for the error of the measurements. This lost heat is shown in Figure 1.

2.2. A procedure that can correct the regular error of structure function based material parameter measurements

A simplified case is discussed first, when the parasitic heat-flow is considered with a single *R* thermal resistance while measuring a *Z*(*s*) thermal impedance. This *Z*(*s*) impedance is constituted by the "ideal" $Z^*(s)$ thermal impedance *to be identified* and the thermal impedance of the shunting heat-flow path.

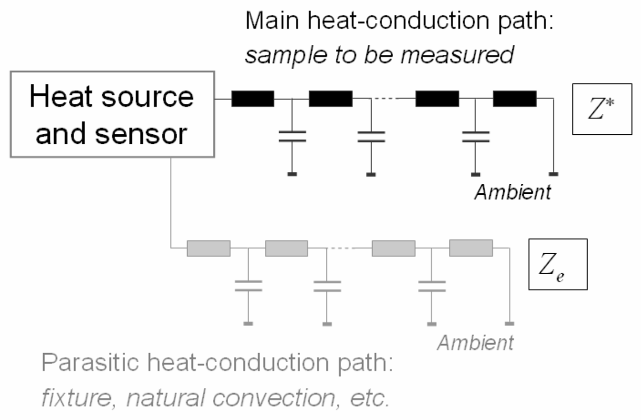

Figure 1: The main (*Z) and the parallel parasitic (*Z*$_e$) heat-flow paths in case of structure function based thermal material parameter measurements**

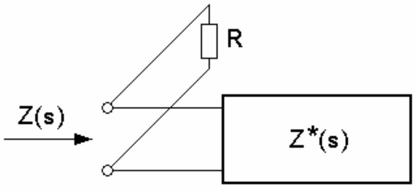

Figure 2:"Ideal" $Z^*(s)$ thermal impedance to be identified together with a shunt thermal resistance *R*

The measured thermal impedance – described by a Foster-model derived directly from the time-constant spectrum – can be written as follows [6]:

$$Z(s) = \sum_{i=1}^{N} \frac{R_i}{1+st_i} \tag{1}$$

where R_i and t_i are the resistance (time-constant magnitude) and time-constant values of the Foster-stages, respectively, $\mathbf{s}=j\omega$, the complex frequency. Rewriting it into a quotient of two polynomials yields

$$Z(s) = \frac{n_0 + n_1 s + n_2 s^2 + ...}{d_0 + d_1 s + d_2 s^2 + ...} = \frac{\sum_{i=0}^{N-1} n_i s^i}{\sum_{i=0}^{N} d_i s^i} \tag{2}$$

where n_i and d_i are the coefficients of the numerator and denominator polynomials, respectively. In case of parallel impedances the effect of *R* can be easily accounted for if the impedances are replaced by their reciprocals:

$$\frac{1}{Z^*(s)} = \frac{1}{Z(s)} - \frac{1}{R} = \frac{\sum_{i=0}^{N} d_i s^i}{\sum_{i=0}^{N-1} n_i s^i} - \frac{G\sum_{i=0}^{N-1} n_i s^i}{\sum_{i=0}^{N-1} n_i s^i} = \frac{\sum_{i=0}^{N-1} (d_i - Gn_i) s^i + d_N s^N}{\sum_{i=0}^{N-1} n_i s^i} \tag{3}$$

where $G = 1/R$. [7] The correction accounting for the effect of *R* is to be carried out in the phase of generating the structure functions, when the thermal impedance is available in the form as given by formula (2) – that is during the Foster-Cauer transformation of the RC model of the impedance [1]. According to (3) the following transformations need to be done in the coefficients of the numerator and denominator polynomials:

$$\boxed{\begin{array}{l} d_i^* = d_i - G \cdot n_i \qquad if \quad i = 0...N-1 \\ d_N^* = d_N \end{array}} \tag{4}$$

If the parallel branch is considered with a $Z_e(s)$ excess thermal impedance function, the corrected structure function may be constructed as follows:

$$\frac{1}{Z^*(s)} = \frac{1}{Z(s)} - \frac{1}{Z_e(s)} \tag{5}$$

where $Z_e(s)$ is the excess thermal impedance that is the source of the error. This is the thermal impedance represented by all the heat paths outside the measured path.

If the value of $Z_e(s)$ is known, based on the the analogy of expression (3) for the purely resistive parasitic heat-flow path, $1/Z^*(s)$ can be represented by

$$\frac{1}{Z^*(s)} = \frac{\left(\sum_0^N d_i s^i\right)\left(\sum_0^{M-1} n_{ej} s^j\right) - \left(\sum_0^{N-1} n_i s^i\right)\left(\sum_0^M d_{ej} s^j\right)}{\left(\sum_0^{N-1} n_i s^i\right)\left(\sum_0^{M-1} n_{ej} s^j\right)} \tag{6}$$

where d_i and n_i represent the denominator and numerator coefficients of the measured impedance, while d_{ej} and n_{ej} represent those of the thermal impedance function describing the measurement error.

The procedure to be used in order to eliminate the systematic measurement error is now as follows:

1. Measure the transient response of the parasitic heat-flow path, construct the structure function, or determine the total steady state thermal conductance of the parasitic path,
2. Measure the transient response of the fixture with the sample, construct the structure function,
3. Make the corrections according to Eq.(3) or Eq.(6).
4. If there are several parasitic heat-flow paths, repeat 1-3 for each of these.
5. Calculate the material parameters from the corrected structure function.

3. Practical implementation of the correction method

3.1. The case of a single parasitic thermal resistance

In case of a purely resistive parasitic heat-flow path formula (3) is to be used. This formula is simply a slight modification of the (2) polynomial representation of the measured thermal impedance. This representation always used in the chain of the $a(z)$? $R(t)$? *Foster model* ? *Cauer model* transformations, needed to obtain the $C_\Sigma(R_\Sigma)$ cumulative and the dC_Σ/dR_Σ differential *structure functions*. After the substitutions given by (4) the standard $a(z)$? *structure functions* conversion procedure can be used. In the transformation procedure the C_Σ or dC_Σ/dR_Σ value at a given R_Σ value of the numerical representation of the structure functions is directly obtained from the n_i and d_i coefficients of the numerator and denominator polynomials of the thermal impedance function. These numerator and denominator polynomials are ill-behaved for any $Z(s)$ thermal impedance.

On one hand, during a typical $a(z)$? *structure functions* conversion procedure the degree of these polynomials is around 150. On the other hand their coefficients (the n_i and d_i values) span over 200-300 orders of magnitude. For this reason the implementations of formulae (2) and (3) need a high precision arithmetics. The evaluation of formula (2) needs 150-200 decimal digits of precision in the mantissa. Since formula (2) can be well represented with such a high precision mathematical library, application of the substitutions given by (4) does not lead to any implementation problem (only the d_i coefficient values are modified).

3.2. Considering a complex parasitic thermal impedance

In expression (6) the products of sums yield polynomials of N+M-1 and N+M-2 degree. If both the $Z(s)$ measured and the $Z_e(s)$ parasitic thermal impedances are accurately represented, formula (6) results in numerator and denominator polynomials of a degree of about 300! Even if M is restricted to 3..15, the distribution of the magnitudes of the coefficients in the numerator of (6)is such that the subtraction during the computer calculations results in false values. This means that structure function values generated from the numerator/denominator coefficients obtained by the direct application of formula (6) may be uselessly erroneous.

For obtaining the corrected $Z^*(s)$ thermal impedance value according to formula (5), both $Z(s)$ and $Z_e(s)$ have to be represented such that no numerical representation problems arise during calculations. This can be obtained in the complex frequency domain. In the complex frequency domain any Z impedance is represented by a set of an R_i and a t_i values, that are the resistance (time-constant magnitude) and time-constant values, as presented in (1):

$$\frac{1}{Z^*(s)} = \frac{1}{\sum_{i=1}^{N} \frac{R_i}{1+st_i}} - \frac{1}{\sum_{j=1}^{M} \frac{R_{ej}}{1+st_{ej}}} \qquad (7)$$

By using (7) we can calculate the corrected thermal impedance using standard IEEE 8-byte (double precision) floating point numbers – represented in the complex frequency (s) domain. Unlike the polynomial representation, the s-domain description of the corrected thermal impedance can not be used for the direct generation of the structure functions. Fortunately, there exists another transformation that converts a thermal impedance given in the complex frequency domain to the time-constant representation [5]:

$$R(z) = \mp \frac{1}{p} \operatorname{Im} \mathbf{Z}(\mathbf{s} = -\exp(-z)) \,. \qquad (8)$$

Once we have the time-constant spectrum available, we can generate the corresponding structure functions as in case of any other thermal impedance.

Equation (8) suggests that the $j\omega$ imaginary frequency has to be replaced only by the $s = -\exp(-z)$ complex frequency, and then the imaginary part of the calculated complex response multiplied by $1/\pi$ provides the time-constant spectrum. In order to avoid poles on the negative real axis in the complex plane the numerical implementation of formula (8) is not straightforward – for details refer to [5].

The correction procedure *in practical implementation* is as follows:

1. Measure the transient response of the parasitic heat-flow path, construct the structure function, or determine the total steady state thermal conductance of the parasitic path,
2. Measure the transient response of the fixture with the sample, construct the structure function,
3. If the effect of the parasitic heat-flow path can be well described with a lumped thermal resistance,
    - correct the coefficients of polynomials according to formula (4)

  - and identify the structure function of the corrected thermal impedance function with the standard procedure, continued from the polynomial representation of the thermal impedance.
4. If the parasitic heat-flow path is far more complex than a single, lumped thermal resistance do the following:
   - calculate the $Z^*(s)$ corrected thermal impedance in the complex frequency domain with $s = -\exp(-z)$, according to Eq.(7) by using Foster representation of the measured and parasitic thermal impedances,
   - convert the $Z^*(s)$ function to its time-constant representation based on Eq.(8), using the numerical procedure described in [9],
   - perform the standard structure function generation procedure starting from the time-constant spectrum representation.
5. If there are several parasitic heat-flow paths, repeat steps 1-4 for each of these.
6. Calculate the material parameters from the corrected structure function.

4. Application examples

To demonstrate the importance of the correction method two examples are presented: the application of the correction in the measurement of the effective thermal conductivity of printed circuit boards, and in the measurement of small thermal resistance values.

In order to ease understanding the examples, we restrict ourselves to considering the parasitic heat-flow paths as purely resistive ones. Examples for subtracting complex heat-flow paths like elimination of a parasitic DCP4 path from e.g. a DCP1 result of a PROFIT/DELPHI thermal transient measurement will be presented elsewhere.

4.1. Example 1: Measuring the effective thermal conductivity of patterned PCB-s

The measurement of the effective thermal conductivity of boards is based on the evaluation of the *cumulative structure function*. The board to be measured is inserted in a fixture that is based on an isothermal ring (see Figure 3). A transistor in the heater/sensor head acts both as heater and sensor. This arrangement assures radial heat-flow in the board between the tip and the isothermal ring. From the thermal transient of the heating/sensing transistor captured by the thermal transient tester equipment the cumulative structure function is derived. The section of the structure function that corresponds to the radial spreading in the PCB is a straight-line segment next to the singularity, at the end of the function 4.

In this measurement the natural convection taking place at the board surface causes a parallel heat-flow path on one hand, on the other hand the a parallel path is generated by the measuring fixture itself, via the powering structure as shown in Figure 4.

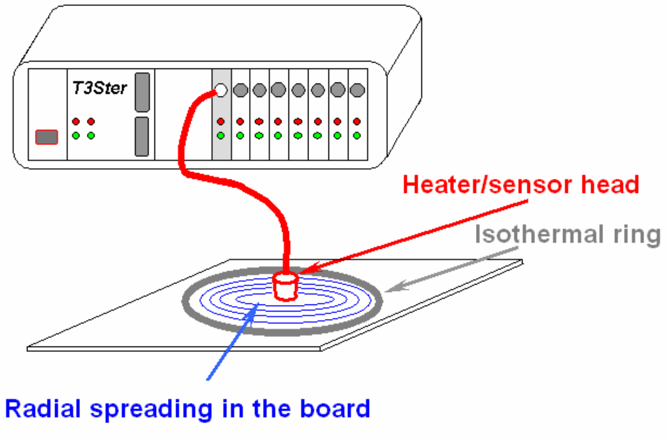

Figure 3: Schematic of the setup for measuring board thermal conductivity

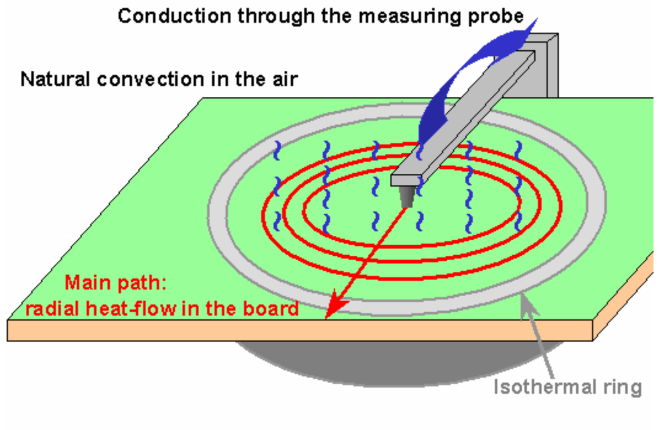

Figure 4: Parasitic parallel heat-flow paths in the effective board thermal conductivity measurement setup

The effect of all these can be lumped into a single shunting conductance. Both effects can be exactly identified if a calibration board is placed into the fixture and is measured in vacuum and in a still air chamber: the shunting conductance in the latter case is the correction value to be considered in all subsequent measurements of other boards. This shunting conductance represents both the conduction through the fixture and the heat-loss through natural convection.

In the following example the correction is done with the effect of the parasitic heat-flow upward in the fixture only. The correction with the losses caused by the natural convection needs a vacuum chamber, but can be done similarly, and will be done in the final version of the paper.

In Figure 6 the differential structure function of the fixture is presented, measured without a sample. The value of the parallel heat conduction represented by the fixture itself is the reciprocal of the (257 K/W) steady state thermal resistance of the fixture, readable at the right hand side end of the function, that is *G*=3,89 mK/W. This value was used in the correction formula of Eq. (4) for several measured samples, see Table 1.

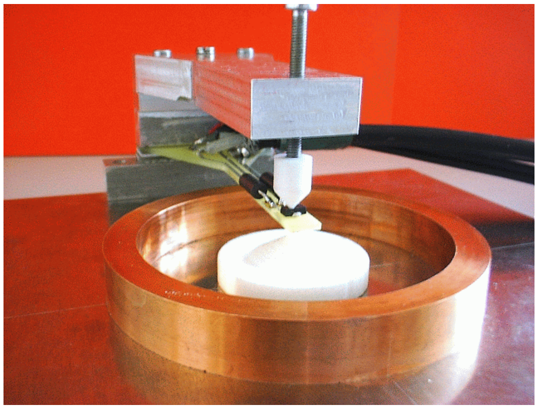

Figure 5: The fixture in reality

	Measured effective thermal conductivity	Corrected effective thermal conductivity	Change in the value, resulted by the correction
samples	w*lambda [W/K]	w*lambda [W/K]	%
Sample_4a	0.01273	0.00889	30
sample_4b	0.01222	0.00846	30
sample_4d	0.01227	0.00870	29
Sample_6_via	0.01370	0.00955	30

Table 1: Results of effective board thermal conductivity measurements[1]

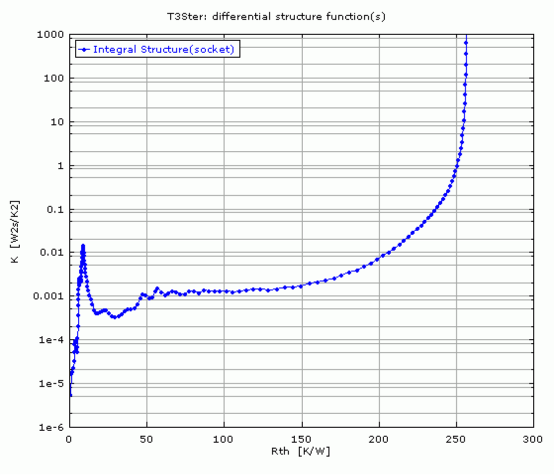

Figure 6: the differential structure function of the fixture without the sample. The value of the parallel heat conduction represented by the fixture itself can be read from the right hand side and of the figure: 1/257 W/K ˜ 3,89*10^{-3} K/W

The structure functions of the sample of the last row of Table 1 are presented in Figure 7. As it is shown in the table and in the figure the correction results in this case in an about 30% modification of the measured result, eliminating the error caused by the parallel heat-flow paths.

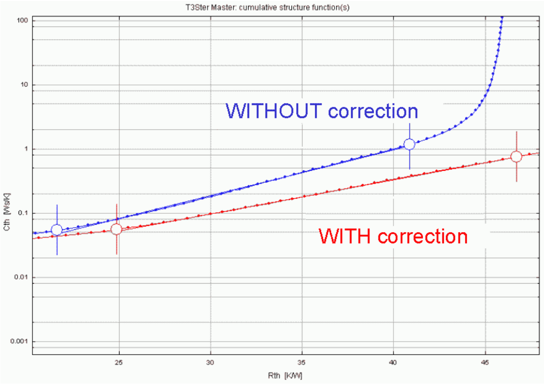

Figure 7: Enlarged details of the structure functions with and without correction, for the sample presented in the last row of Table 1.

5. Example 2: Measurement of thermal resistance of interface materials

The measurement of the thermal resistance of interface materials [2], [3] is based on the use of the *differential structure functions*. In the methodology, presented in [2] the sample to be measured is placed in a fixture, which has two separable parts, within which the sample to be measured has to be placed. The fixture itself is such, that it produces two well distinguishable sharp peaks in the differential structure function, and the sample to be measured has to be placed between the surfaces, represented by these peaks.

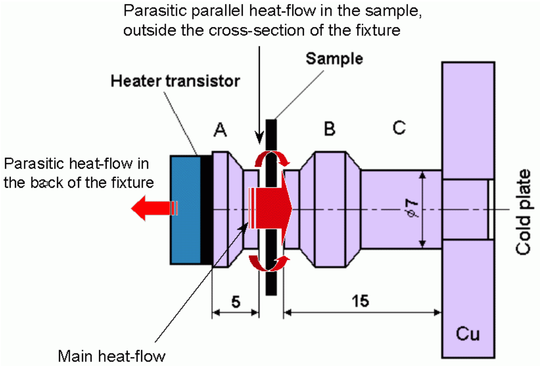

Figure 8: Fixture for measuring thermal resistance of interface materials, indicating possible parasitic heat-flow paths.

During the measurement first the reference function of the empty fixture is identified, then the one with the sample. The measurement principle is based on the shift of peaks in the differential structure function of the fixture as illustrated by Figure 8 and Figure 9.

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[1] In the table the sample_4 samples were all measured on a board of 470µm, sample_6 on a board of 1510µm thickness, patterned on both surfaces, measured at different locations. The sample_6_via contains a thermal via. *w* is the thickness of the samples.

The peak corresponding to section **B** shifts with respect to the peak belonging to section **A** as a sample is inserted into the fixture. The physical arrangement, together with the main and parasitic parallel heat-flow paths is also shown in Figure 8. All the heat, that leaves the transistor and arrives to the ambient on a path, which is different from the main path across the sample, is a loss for the measurement and has to be considered in the evaluation.

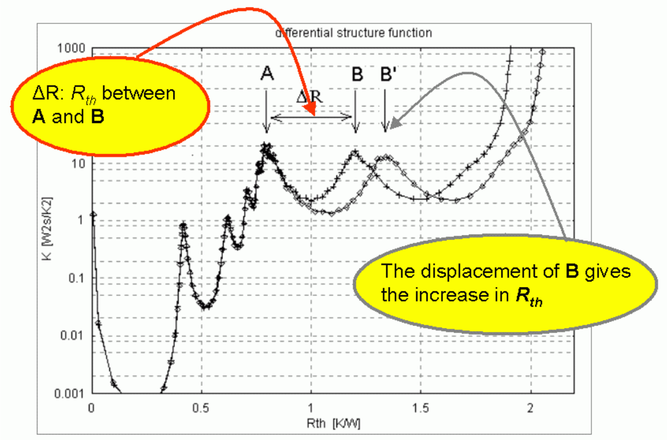

Figure 9: Displacement of peak B in the differential structure function gives the thermal resistance of the sample

The losses can be accounted for if we measure the thermal resistance represented by the parasitic heat-flow without the fixture, and correct the structure function with this value according to the procedure described before. The convection losses can be also calculated with, if we measure the structure once in vacuum [7].

The measured and the corrected results are presented in Figure 10. As we can see from this figure the correction for the parallel losses results in an about 18% increase in the measured thermal resistance value.

6. Conclusions

In this paper we demonstrated, that it is possible to subtract the thermal impedance of shunting branches from structure functions. This will enable the development of software tools for separating different 1D structure function projections of complex 3D heat flow paths, like an IC package.

The method can be used to eliminate the effect of the systematic measurement errors, present in all the cases when we use the structure function based evaluation to determine thermal material parameter values from simple thermal transient measurements.

The advantage of the method is, that the necessary measurements to the correction have to be done only once for a given fixture, and the corrections after this can be done automatically by the measurement evaluation software, if the measurements are done in the same conditions.

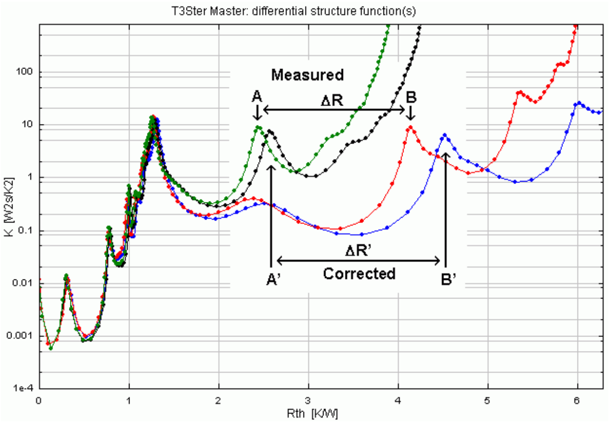

Figure 10: The corrected structure function of the thermal resistance meter

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