B1 NOTE B1.98.5

January 30, 1998

Measuring Coefficient of Thermal Expansion of a Composite C-profile

Kari Tammi

Helsinki Institute of Physics

Abstract

This document describes the measurement system and the measurement data analysis of measuring a composite profile's coefficient of thermal expansion. The Measured C-profile is a frame structure of to the MSGC rod. The C-profile's coefficient of thermal expansion (CTE) is measured by strain gages. The results are the CTEs for the C-profile and a study of errors.

Contents

1 Introduction

Under a thermal load, the material will change shape proportional to the amount of temperature change multiplied by its coefficient of thermal expansion. The coefficient of thermal expansion (CTE) indicates how much the shape of the material will change for each degree of temperature change. CTE can be assumed to be linear in small temperature regions. These measurements are based on this assumption.

The measured object is a composite C-profile. The C-profile is used as a frame in the MSGC rod. The C-profile is made of carbon fibre by pultrusion. The fibres in the C- profile are all oriented longitudinally.

1.1 Different possibilities for the measuring the CTE

1.1.1 Measuring the length

By measuring a beam's length at different temperatures it is possible to calculate the CTE. However an accurate arrangement is quite difficult to build. There are already available that kind of applications. At the Technical Research Centre of Finland is so called dilatemeter that could have been used for this application. Also interferometers are suitable.

In a dilatemeter there is a cylinder in which the object is placed. Then a temperature cycle is run and the length of the object is measured at different temperatures. The dilatemeter is probably more accurate than a strain gage application. However there were some weaknesses that rejected the use of the dilatemeter. With a dilatemeter and an interferometer CTE can be measured only in one direction. There would be also problems with fitting the C-profile into dilatemeter's cylinder. The C-profile wouldn't fit in the cylinder as a whole piece.

1.1.2 Measuring the strain

Another possibility is to measure the strain with strain gages. With multiple strain gages the strain can be measured in every direction.

Strain gages have matched temperature characteristics. In other words, the strain gage's CTE is usually the same as the CTE of the measured material. For usual measurements this is desirable because rough stress and strain calculations don't need a temperature correction. Now it was a bit unwanted because strain gages showed now an "apparent strain". The "Real strain" can still be calculated from the difference between the "apparent strain" and the strain gage's "free strain". The strain gage has also non-ideality, which means that the output is function of the temperature. This is usually also called the "apparent strain". Now it is called a "correction strain" as a more descriptive term in this case.

The problems of this system are inaccuracy in strain gages and the voltage measurement over gages. Strain gages have always some error in CTE, resistance and gage factor. Strain gages have also transverse sensitivity and they might be glued in a wrong angle. Generally length-measuring-applications are probably more accurate, but as said length-measuring-applications measure CTE only in one direction and they don't indicate bending.

2 Measurements

2.1 Arrangement

The resistance is measured by 4-wire coupling. Two of the wires are for a constant current source and the rest two wires are for a digital multimeter. The multimeter used was HP 3458A. Data was acquisited by an HP 3495 scanner and a PC-computer through an HP-IB bus. These equipment were provided by the of Laboratory of Strength of Materials at the Helsinki University of Technology.

According to the HP's specifications [1], the multimeter's input impedance is very high (>10GΩ) and the resistance over the strain gage is 120Ω. Thus it can be assumed that all the current flows through the strain gage. The linearity for the HP 3458A is specified at 0.1 ppm [2] ("parts per million" equals 10^{-6}).

The strain gages are installed in an array. There is also a reference resistor in the array. The reference resistor is a Vishay S102 120Ω. According to the Vishay's specifications the value of the resistance changes (0±1) ppm/°C [3]. The reference resistor is at a constant temperature during the measurements. The reference resistor indicates the changes of the current (Fig. 1). The current doesn't remain exactly constant. It changes and can be corrected in the calculations by voltage over the reference resistor [4].

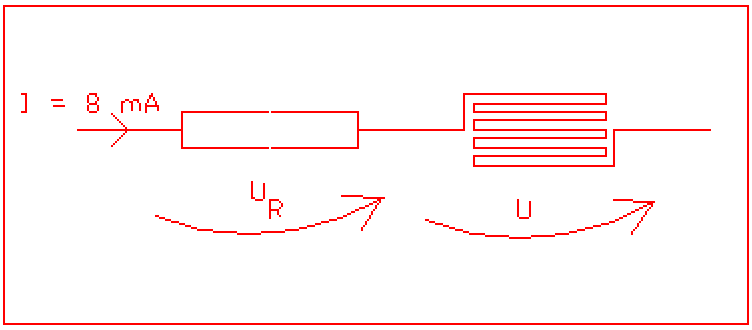

Figure 1: Reference resistor and strain gage.

Multiple strain gages were attached to get a statistical confidence. Seven stain gages were glued longitudinally to the C-profile and three strain gages transversally (Fig. 2). Two strain gages were glued to steel and one to aluminium to verify the calculations (Fig. 3). The strain gages were type KFG-5-120-C1-11L3M3R of the Kyowa Electronic Instruments CO, LTD.

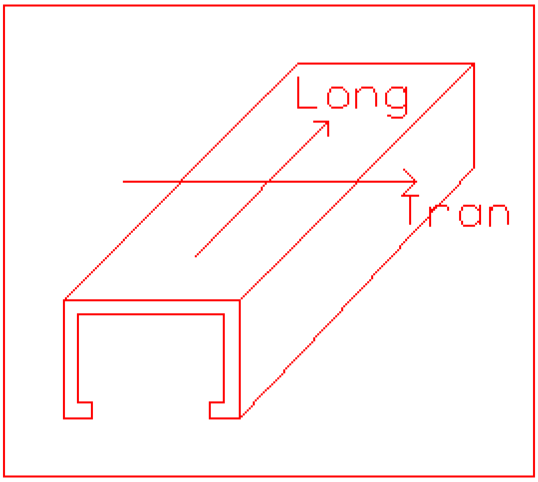

Figure 2: Longitudinal and transversal directions on C-profile.

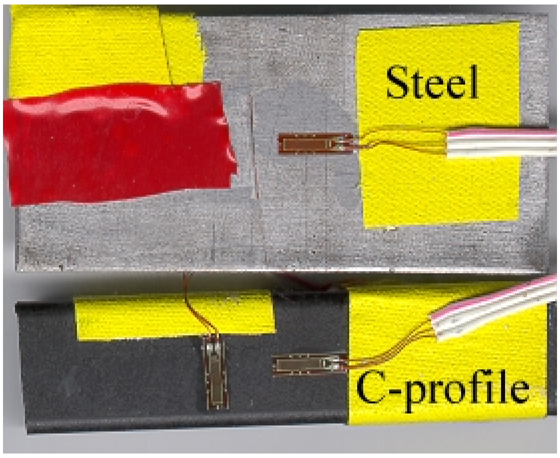

Figure 3: Strain gages glued to steel and C-profile.

An oven was used for heating. The objects were put into a well-isolated box in the oven with a temperature transducer and the temperature cycle was run. There were also some big metal objects in the box to increase the stability of the temperature. When the temperature had reached a stationary state, the measurements were performed and the oven was heated to another temperature.

2.2 Calculations

The relative change of resistance $\frac{\Delta R}{R}$ is proportional to the relative change of length, strain ε, multiplied by a gage factor k:

$$\frac{\Delta R}{R} = k\frac{\Delta L}{L} = k\varepsilon \quad (1)$$

The relative change of voltage is equal to the relative change of resistance. The strain can be calculated roughly by the solving strain from (1):

$$\varepsilon = \frac{\Delta R}{kR} = \frac{U - U_0}{kU_0} \qquad (2)$$

In equation (2) U is the measured voltage after heating and U_0 is the voltage before the heating. If the current doesn't remain constant, we can correct the voltage by multiplying with the relation of the reference voltage. U_{RO}

is the reference voltage at the begin of the measurements and U_R is the reference voltage at each measurement temperature. The corrected voltage is now $U^{'}$ [4]:

$$U^{'} = \frac{U_{R0}}{U_R} U \qquad (3)$$

And now we can obtain a more accurate strain from [4]:

$$\varepsilon^{'} = \frac{1}{k}\left(\frac{U}{U_0}\frac{U_{R0}}{U_R} - 1\right) \qquad (4)$$

According to Kyowa's specifications the "correction strain" (usually called apparent strain) ε_c is a function of the temperature t [^{o}C]. ε_c is in ppms.

$$\varepsilon_C = -26 + 2.3t - 0.056t^2 + 0.00033t^3 - 0.00000035t^4 \qquad (5)$$

The corrected strain ε is:

$$\varepsilon = \varepsilon^{'} - \varepsilon_C \qquad (6)$$

With Equations (4) and (5) we can evaluate the "apparent strain" in Equation (6). After that we can calculate how much the strain gage would extend freely. This strain is called the "free strain". The "Real strain" is now the difference between the "apparent strain" and the "free strain". Finally we can calculate CTE by dividing the difference by the temperature difference. In Equation (7) α is the unknown CTE, α_{SG} is the strain gage's CTE. ΔT is the temperature difference the between start and the end temperatures, in other words the temperature cycle. It may be a bit confusing to add the term $\alpha_{SG}\Delta T$ to the "apparent strain". This is done because the "apparent strain" is now negative and we want to get difference. (A person who is familiar with strain gages may feel a bit confused because now the "apparent strain" has different meaning than usual.)

$$\alpha = \left[\frac{1}{k}\left(\frac{U}{U_0}\frac{U_{R0}}{U_R} - 1\right) - \varepsilon_C + \alpha_{SG}\ \Delta T\right]\frac{1}{\Delta T} \qquad (7)$$

There is also another way to calculate the CTE. If there is an available reference material that has an exactly known CTE, the unknown CTE can be calculated from the strain difference between the known and the unknown materials. Strains can be calculated:

$$\varepsilon = (\alpha - \alpha_{SG})\Delta T \qquad (8)$$

$$\varepsilon_{ref} = (\alpha_{ref} - \alpha_{SG})\Delta T \qquad (9)$$

Equation (8) gives strain for the unknown and Equation (9) for the known reference material. From (8) and (9) α can be solved [5]:

$$\alpha = \alpha_{SG} + \frac{\varepsilon}{\Delta T} = \alpha_{ref} + \frac{\varepsilon - \varepsilon_{ref}}{\Delta T} \qquad (10)$$

The strain gage's "correction strain" doesn't have effect now as we can see from (10). This is also a used method to measure CTE [5].

2.3 Errors

2.3.1 Differentials

We can calculate the error for Equation (7) with a differential:

$$\Delta\alpha = \frac{\partial\alpha}{\partial x_i}\Delta x_i \approx \Delta\alpha_{SG} \qquad (11)$$

The error of the strain gage's CTE shows to be the dominating term. It is about 10 times greater than the other terms. There is no exact value for this error, but it is between (0.1..1) μm/°C. Kyowa guarantees 1 μm/°C for every strain gage, but strain gages from same process are much more accurate [5].

The error for Equation (7) is:

$$\Delta\alpha = \frac{\partial\alpha}{\partial x_i}\Delta x_i \approx \Delta\alpha_{ref} \qquad (12)$$

Now the error of the known material's CTE is dominating. In this case the error is easily larger than in the first case, but we can get a confirmation for calculations.

2.3.2 The transverse sensibility and the angle error

Some error sources are invisible in the differentials Equations (11) and (12). Strain gages can be misaligned in a wrong angle and the strain gage has (-0.17 ± 0.49) % transverse sensitivity. These errors are usually quite small and can be neglected in most applications. Let us assume that the longitudinal CTE is -0.3 µm/°C, the transversal CTE is 30 µm/°C and the strain gages are glued in a wrong angle between -1° and 1°. These errors are important only in calculating the longitudinal CTE, because then the more expansive direction is perpendicular to the strain gage (Fig. 4).

The transverse sensitivity will affect on the standard deviation of measurements and the error is included in the standard deviation. There is also a possibility to correct the error by calculations. The correction can be done by subtracting ε_{ts} from the "real strain", which describes that perpendicular strain affects -0.17% on the strain gage's output when temperature difference is ΔT:

$$\varepsilon_{ts} = \frac{-0.17\%}{100\%} 30 \cdot 10^{-6} \frac{1}{{}^{o}C} \Delta T \qquad (13)$$

The angle error has also a remarkable influence, because the absolute value of the transversal CTE is 10 times the absolute value of the longitudinal CTE. The angle error affects on the results by making the CTE more positive, because the transversal CTE is positive. An average value of the absolute angle is 0.5°. Error in the strain ε_a is calculated with the following equation when the strain gages are glued in a 0.5° angle:

$$\varepsilon_a = \left(-0.3\cos 0.5^o + 30\sin 0.5^o\right)\Delta T \cdot 10^{-6} \frac{1}{{}^{o}C} \approx (-0.30 + 0.26)\Delta T \cdot 10^{-6} \frac{1}{{}^{o}C} \qquad (14)$$

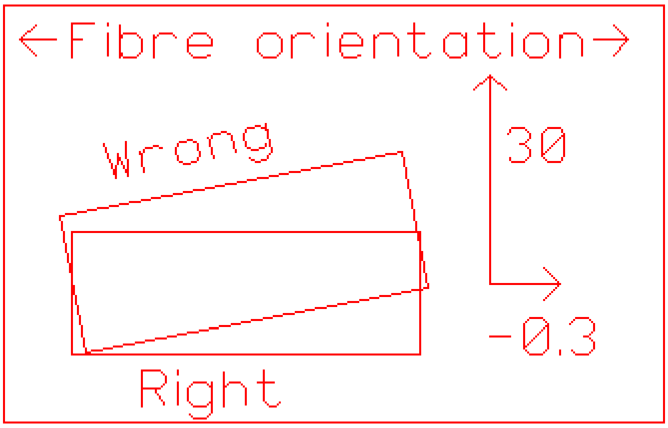

Figure 4: The strain gage in the right and wrong angle. Fibre orientation is horizontal.

From Equation (14) can be seen that a strain gage has almost zero output if it is misaligned by 0.5° . After an examination it showed up that this was a realistic average error for angle. This is a source of systematic error if not corrected. Errors (13) and (14) can be combined:

$$\varepsilon_{tot} = 30\left(\sin 0.5^o - 0.0017\right)\Delta T \cdot 10^{-6} \frac{1}{{}^{o}C} \qquad (15)$$

The term ε_{tot} is subtracted from the "real strain" before the CTE is calculated from Equation (7).

3 Results

As a result of the calculations and the study of errors, the longitudinal CTE is obtained from:

$$\alpha_{par} = \left[\frac{1}{k}\left(\frac{U}{U_0}\frac{U_{R0}}{U_R} - 1\right) - \varepsilon_C - \varepsilon_{tot} + \alpha_{SG}\ \Delta T\right]\frac{1}{\Delta T} \qquad (16)$$

The CTE perpendicular to the symmetry axis isn't sensitive to transverse sensitivity or angle error. It is obtained from:

$$\alpha = \left[\frac{1}{k}\left(\frac{U}{U_0}\frac{U_{R0}}{U_R} - 1\right) - \varepsilon_C + \alpha_{SG}\ \Delta T\right]\frac{1}{\Delta T} \qquad (7)$$

The temperature cycle was run from 16.6 °C to 38.1 °C and to 66.5 °C. The voltage measurements were done five times at each temperature. Previous measurements were done with a greater amount of voltage measurements. Standard deviation remained on the same level even if the amount of voltage measurements was reduced. The temperature was stable during the final measurements and the deviation of the voltages remained constant.

The selection of the temperature cycle was difficult, because a large cycle is better for accuracy, but there can also be some non-linearity. It seems that the longitudinal CTE becomes a bit more negative, when the temperature increases. The CTE close to the room temperature is adequate in this case.

Strain gages were glued to the flange and to the waist of the C-profile to indicate bending. No indications for bending were found. Thermal expansion of the flange and the waist seemed to be equal.

As a result it can be concluded that at room temperature the longitudinal CTE is (-0.5 ± 0.4) μm/°C and the transversal CTE is (30 ± 2) μm/°C.

4 Discussion

The same results were also obtained by calculating with the reference material (Equation (10)). The deviation was larger in that case, but it gave confirmation to the previous results. More confirmation could be achieved by calculating already known CTEs. The CTEs for steel and aluminium matched quite good to the known values. The steel measurements gave a CTE of 12 μm/°C and pure aluminium 23 μm/°C.

From material tables we can get some known values [6]. For XA-fibres the longitudinal CTE is -0.26 μm/°C and the transversal CTE 26 μm/°C. For HM-fibres the values are -1.3 μm/°C (longitudinal) and 25 μm/°C (transversal). The measured longitudinal CTE is between those values, while the transversal CTE is slightly bigger.

The error limits are taken from the standard deviation in measurements and rounded upwards. This is done because all remarkable errors (strain gage's CTE, transverse sensitivity and angle error) are included in the standard deviation. If there were also remarkable errors that are "invisible" in the standard deviation, those errors should be added to the standard deviation.

The error, 0.4 μm/°C, is very big compared to the absolute value 0.5 μm/°C, but 0.4 is μm/°C already close to the minimum value of error of the strain gage's CTE. Strain gages may not produce the smallest error. Other applications could be more accurate, especially when the CTE's absolute value is under 1 μm/°C. Nevertheless every material has a deviation and CTE is function of temperature. CTEs are always some sort of average values.

5 Acknowledgements

I would like to thank Dr. Tech. Miikka Kotamäki for advice and a interesting research subject and MSc Iikka Järvenpää for very useful advice on the theoretical and the practical field and a possibility to use the measuring equipment. My thanks are also due to Engineer Kari Kukkurainen for a competent product support on the field of strain gages.

6 References

[1] Hewlett-Packard. Hewlett-Packard 3458A Multimeter, Operating, Programming and Configuration Manual, Edition 1. USA, 1988.

[2] Hewlett-Packard. Hewlett-Packard Journal. Technical Information from the Laboratories of Hewlett-Packard Company. Volume 40, Number 2, April 1989. Holland 1989. 99p.

[3] Vishay Resistor Products. Short from Catalog. Bulletin SF-100.

[4] Discussions with M.Sc. Iikka Järvenpää. Laboratory Manager, Laboratory of Strength of Materials, Helsinki University of Technology.

[5] Discussions with Engineer in Measurement and Control Kari Kukkurainen. Sales Consultant. Honeywell LTD, Finland.

[6] Ilkka Airasmaa, Juha Kokko, Veikko Komppa, and Olli Saarela. Muovikomposiitit. Muoviyhdistys Ry, Gummerrus Kirjapaino Oy, Jyväskylä, 1991. 575 p.

7 Appendix 1: Temperature corrections

Corrections to the strain and gage factor according to the Kyowa's specifications (Kyowa's KFG-5-120-C1-11L3M3R strain gages). Graphs of temperature dependence (Fig. 1 and Fig. 2) and tables to calculate values are shown here.

The strains are calculated at different temperatures and the "correction strain" is the difference between the strains (Table 1). Correction strain graph follows equation:

$$\varepsilon_C = -26 + 2.3t - 0.056t^2 + 0.00033t^3 - 0.00000035t^4 \qquad (1)$$

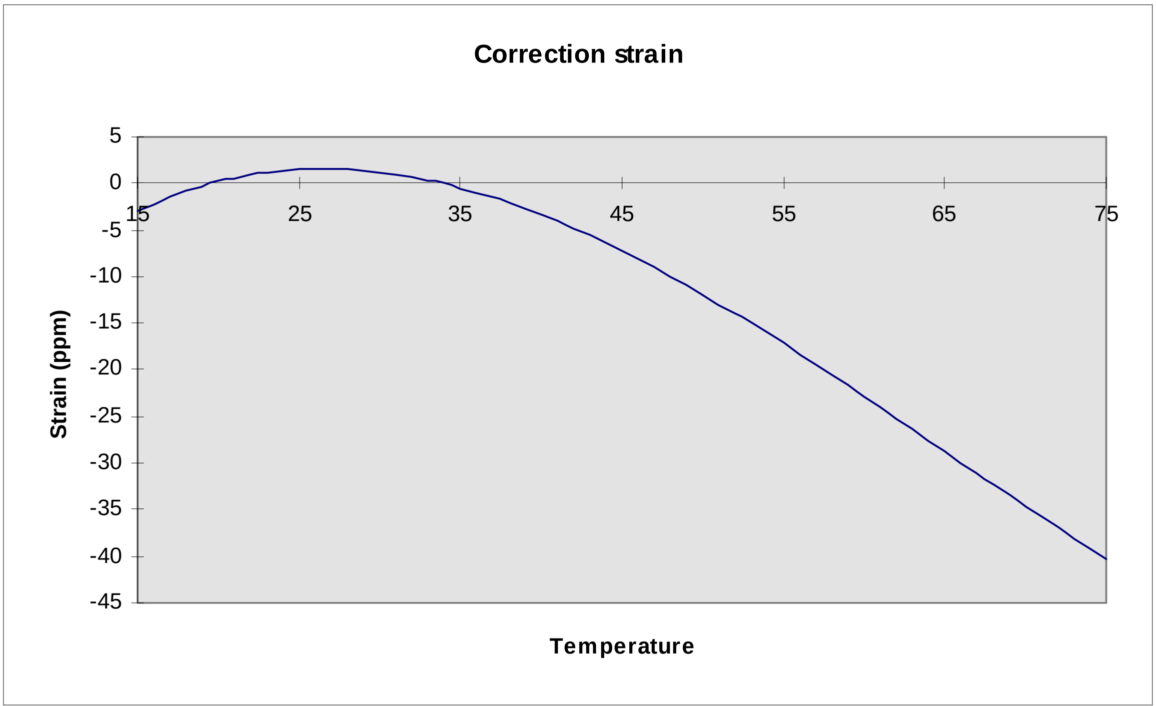

Figure 1: The "correction strain" as a function of temperature.

Table 1: The "correction strain" (in ppms) calculated at the measuring temperatures.

T1	16.6	-1.76842
T2	38.1	-2.14658
Correction strain		-0.3782

The gage factor is also a function of temperature, but very slightly (Fig. 2). Gage factors are also calculated at measuring temperatures (Table 2).

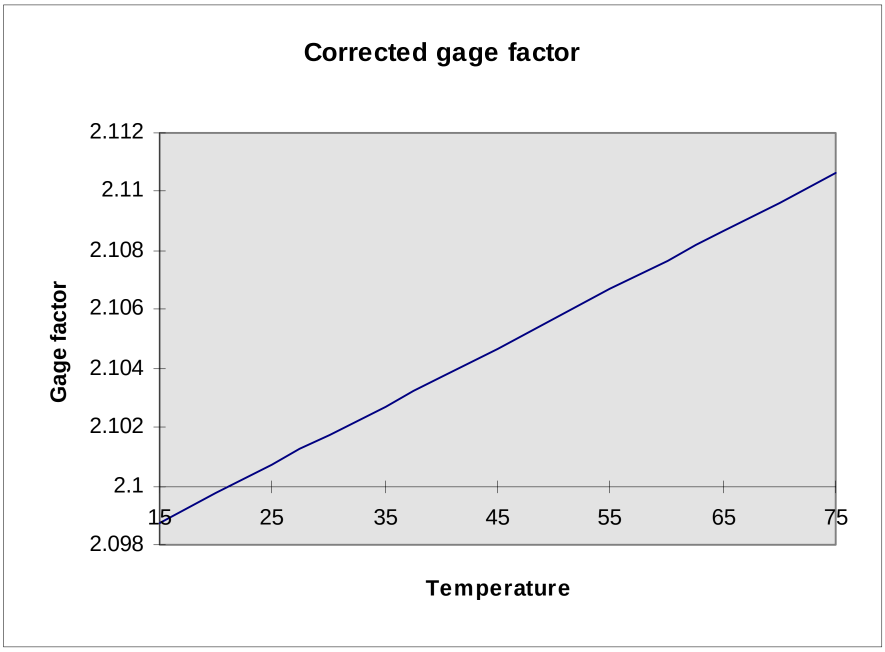

Figure 6: The gage factor as a function of temperature.

Table 2: The gage factor calculated at measuring temperatures.

T	Gage Factor
16.6	2.0991
38.1	2.1033

8 Appendix 2: Example to calculate CTE

An Example to calculate the CTE from measurement data. Average values are calculated from the voltage measurements.

$$\alpha = \left[\frac{1}{k} \left(\frac{U}{U_0} \frac{U_{R0}}{U_R} - 1 \right) - \varepsilon_C + \alpha_{SG} \ \Delta T \right] \frac{1}{\Delta T} \qquad (1)$$

$$\alpha_{par} = \left[\frac{1}{k} \left(\frac{U}{U_0} \frac{U_{R0}}{U_R} - 1 \right) - \varepsilon_C - \varepsilon_{tot} + \alpha_{SG} \ \Delta T \right] \frac{1}{\Delta T} \qquad (2)$$

The calculations follow Equation (1) with transverse sensitivity and angle error correction (2) as described before. From voltage measurements is calculated average values. The strains are corrected and the "real strains" got. The "free strains" are added. Strains from the transverse sensibility and the angle error are subtracted when the longitudinal CTE is calculated. This is called the "Transverse Correction". Finally the CTEs are got by dividing the free strains by the temperature cycle. Average values with standard deviations are in the box.

Calculation example

CTE for strain gage	11.7							
		Ref	Parallel	Parallel	Perp	Steel 1	Steel 2	Alum
Temperature	16.6	1.016794	1.016914	1.016713	1.016401	1.015713	1.01509	1.016094
Corrected strain [ppm]	0	1.016795	1.016913	1.016712	1.016402	1.015713	1.015091	1.016094
Corrected gage factor	2.0991	1.016795	1.016912	1.016712	1.016403	1.015714	1.01509	1.016095
		1.016795	1.016911	1.01671	1.016405	1.015714	1.01509	1.016096
		1.016794	1.01691	1.016709	1.016408	1.015713	1.015091	1.016096
Average Voltage		**1.016795**	**1.016912**	**1.016711**	**1.016404**	**1.015713**	**1.01509**	**1.016095**
Stdev		**5.47E-07**	**1.58E-06**	**1.64E-06**	**2.77E-06**	**5.48E-07**	**5.48E-07**	**1E-06**
Temperature	38.1	1.016797	1.016385	1.016174	1.017208	1.015731	1.015093	1.016628
Corrected strain [ppm]	-0.378	1.016796	1.016383	1.016174	1.01721	1.015729	1.015095	1.016629
Corrected gage factor	2.1033	1.016796	1.016383	1.016173	1.017212	1.015731	1.015093	1.016631
		1.016796	1.016383	1.016172	1.017213	1.015731	1.015093	1.016632
		1.016796	1.016382	1.016171	1.017213	1.015731	1.015094	1.016633
Average Voltage		**1.016796**	**1.016383**	**1.016173**	**1.017211**	**1.015731**	**1.015094**	**1.016631**
Stdev		**4.47E-07**	**1.1E-06**	**1.3E-06**	**2.17E-06**	**8.95E-07**	**8.94E-07**	**2.07E-06**
Real Strain			-0.00025	-0.00025	0.000377	7.68E-06	1.13E-06	0.00025
Free strain			0.000252	0.000252	0.000252	0.000252	0.000252	0.000252
Transverse Correction			4.53E-06	4.53E-06				
CTE			**-2.7E-08**	**-2.4E-07**	**2.92E-05**	**1.21E-05**	**1.18E-05**	**2.33E-05**

		Parallel	Parallel	Parallel	Parallel	Parallel	Perp
Temperature	16.6	1.016193	1.016519	1.015372	1.015065	1.015899	1.014102
Corrected strain [ppm]	0	1.016192	1.016517	1.015371	1.015065	1.015896	1.014105
Corrected gage factor	2.0991	1.016191	1.016516	1.01537	1.015063	1.015896	1.014107
		1.016189	1.016516	1.015367	1.015063	1.015895	1.01411
		1.016189	1.016514	1.015368	1.015062	1.015892	1.014115
Average Voltage		**1.016191**	**1.016516**	**1.01537**	**1.015064**	**1.015896**	**1.014108**
Stdev		**1.79E-06**	**1.82E-06**	**2.07E-06**	**1.34E-06**	**2.51E-06**	**4.97E-06**
Temperature	38.1	1.01565	1.015952	1.01483	1.014508	1.015333	1.015013
Corrected strain [ppm]	-0.378	1.01565	1.015949	1.014829	1.014508	1.015332	1.015015
Corrected gage factor	2.1033	1.015648	1.015948	1.014826	1.014506	1.015331	1.015018
		1.015646	1.015947	1.014826	1.014504	1.015329	1.015019
		1.015645	1.015946	1.014825	1.014505	1.015327	1.01502
Average Voltage		**1.015648**	**1.015948**	**1.014827**	**1.014506**	**1.01533**	**1.015017**
Stdev		**2.28E-06**	**2.3E-06**	**2.17E-06**	**1.79E-06**	**2.41E-06**	**2.92E-06**
Real Strain		-0.00025	-0.00027	-0.00025	-0.00026	-0.00026	0.000426
Free strain		0.000252	0.000252	0.000252	0.000252	0.000252	0.000252
Transverse Correction		4.53E-06	4.53E-06	4.53E-06	4.53E-06	4.53E-06	
CTE		**-3.4E-07**	**-8.8E-07**	**-3.4E-07**	**-6.7E-07**	**-8.3E-07**	**3.15E-05**

CTE	**Par**	**Perp**
Average	-4.768E-7	2.956E-5
Stdev	3.226E-7	1.808E-6