

## ABSTRACT

Edge/end effects on the Coefficient of Thermal Expansion (CTE) of a sandwich structure with carbon fiber reinforced plastic (CFRP) facesheets and an aluminum honeycomb core were studied via Finite Element Analysis (FEA), a modified laminate plate theory and measurements by Michelson interferometry and a contactless optical reflectance technique. The four methods agree within +/- 8%. The modified laminate approach accounts for CTE changes with increasing core thickness. The optical reflectance technique verifies the FEA predictions of CTE distribution. Such results guide the positioning of sensors for thermophysical property measurements.

## INTRODUCTION

Free edge, end and size effects have been recognized as critical parameters which control the failure modes of composite materials and structures. Edge effects penetrate further into the structure of a three layer sandwich plate than they would in a homogeneous plate [1]. Routine application of Saint-Venant's principle, which limits the extent of edge disturbances, is not justified in the solution of sandwich structures [2,3]. Horgan [3] summarizes the situation with sandwich structures where the Young's modulus of the core  $E_c$  is small compared to that of the surface layers. When the ratio of Young's modulus to shear modulus is large, end effects are transmitted over a distance which is of the order of several specimen widths. Whitney [4] also pointed out that due to geometrically influenced edge stresses, the elastic response and strength of angle ply laminates becomes a function of sample width. Consequently, test sample dimensions and the means used to measure the strain response may affect the design of mechanical grips. This in turn suggests that the position of sensors or reference points may also be critical for other thermophysical properties, such as thermal and moisture expansion coefficients (CTE and CME).

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This paper analyzes a symmetrical CFRP composite facesheet with an aluminum honeycomb core structure by predicting its CTE distribution via FEA and a modified laminate plate theory and then comparing these predictions with measurements made by Michelson interferometry and also a contactless optical reflectance technique.

## BACKGROUND

FEA has been a major tool in the study of free edge effects on hygrothermal behavior in angle-ply laminates [5-8]. Wang and Crossman [6] showed that a laminate can thicken under a  $1^\circ$  temperature increase near the edge. Farley and Herakovich [7] showed that hygrothermal loadings can significantly alter the free edge stresses and these in turn modify the mechanical behavior. Kural and Ellison [8] used FEA to show that the end of a composite sample is distorted by thermal loading and an error in the CTE measurement occurs if the ends are used. The extent of this error depends on sample dimensions, fiber orientations and materials.

Marchetti and Morganti [9] reported on the use of FEA to predict the CTE of honeycomb sandwich structures. A NASTRAN model with 161 grid points was employed and an end loaded dilatometer was used to verify the predictions. Scolamiero [10] compared cell structure FEA models with analytical and test simulation models and proposed a method for predicting in-plane core stiffness based compression modulus measurements. Chamis et al [11] used FEA to develop a computer code for hygrothermomechanical behavior but no results are given for the specific effects of edges. FEA was also used to model the effect of adhesive in honeycomb structures [12]. In this work the predicted CTE was checked with an optical technique which involved detection of targets mounted on the specimen. However, few workers have reported on the free edge stress analysis of sandwich structures [13].

Analytical models to predict CTE and CME are attractive because of the ease of parametric analysis and relative independence of geometry. Such models [14-17] use laminate plate theory together with models based on honeycomb parameters, such as cell expansion angle and dimensions to obtain equivalent in-plane core stiffnesses. However, a substantial core thickness involves a complex stress state and we shall show that this effect should and can be incorporated into a plate theory. An analytical model for hygrothermal effects with a discontinuous skin is outlined by Frostig [18].

Measurements on the CTE of sandwich structures are reported in [9, 12, 19-22]. A particularly accurate approach is Michelson interferometry with mirrors placed on the surface [19-22] (and where care is generally taken to avoid the ends or edges of sample). An optical reflectance method to measure the CTE at discreet points was introduced in [22].

## FINITE ELEMENT MODEL PREDICTIONS

Table 1 summarizes the input properties of the test sample used. A one-eighth symmetry FEA model using ANSYS [23] software was constructed. Eight-node, quadrilateral shell elements were used to model both the honeycomb and face sheets. The presence of the adhesive was ignored due to its low stiffness and limited contact with the core. A total of 6560 elements were used. Figure 1 shows the one-eighth symmetry model, corresponding to 101.5 x 25.4 x 25.4 mm in the x, y, and z directions, resp. Figure 2 shows the contours of the fairly uniform displacement distribution on the facesheet surface in the x-direction corresponding to a one Kelvin uniform loading. The left and bottom edges in figures 2-5 represent the center (reference) lines. The letters MN and MX point out the minimum and maximum values, resp. Figure 3 also shows there is uniform (local) strain distribution except at the right (free) end. The periodic strain variations show the effect of the honeycomb geometry. Figures 4 and 5 show substantial variation in displacement and strain across the sample. The FEA model also provides 3D displacements and shows that the honeycomb core bulges at the ends and sides on heating. Figure 6 shows the x-direction surface displacement as  $f(z)$  at the sandwich centerline ( $y=0$ ) starting at  $x = 101.5$  mm.

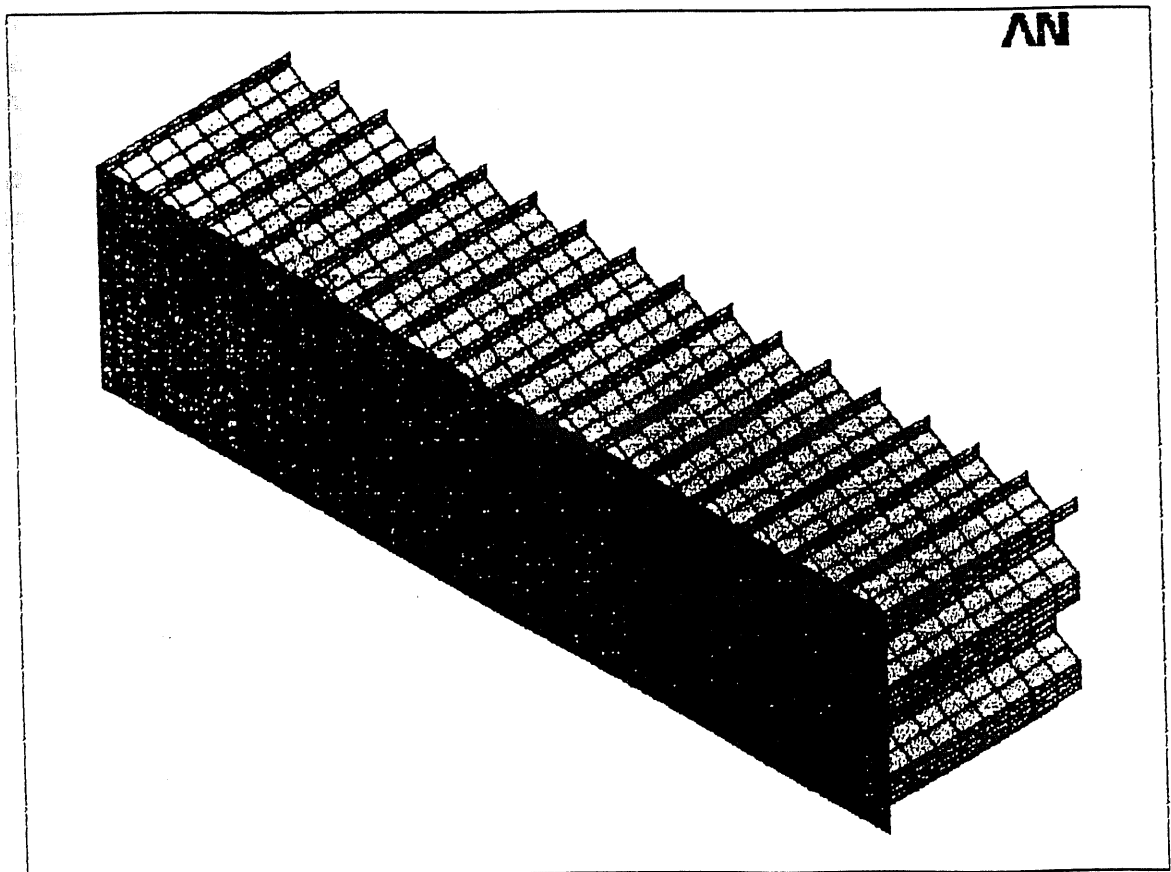


Figure 1 One-eighth symmetry FEA model of facesheet and honeycomb core

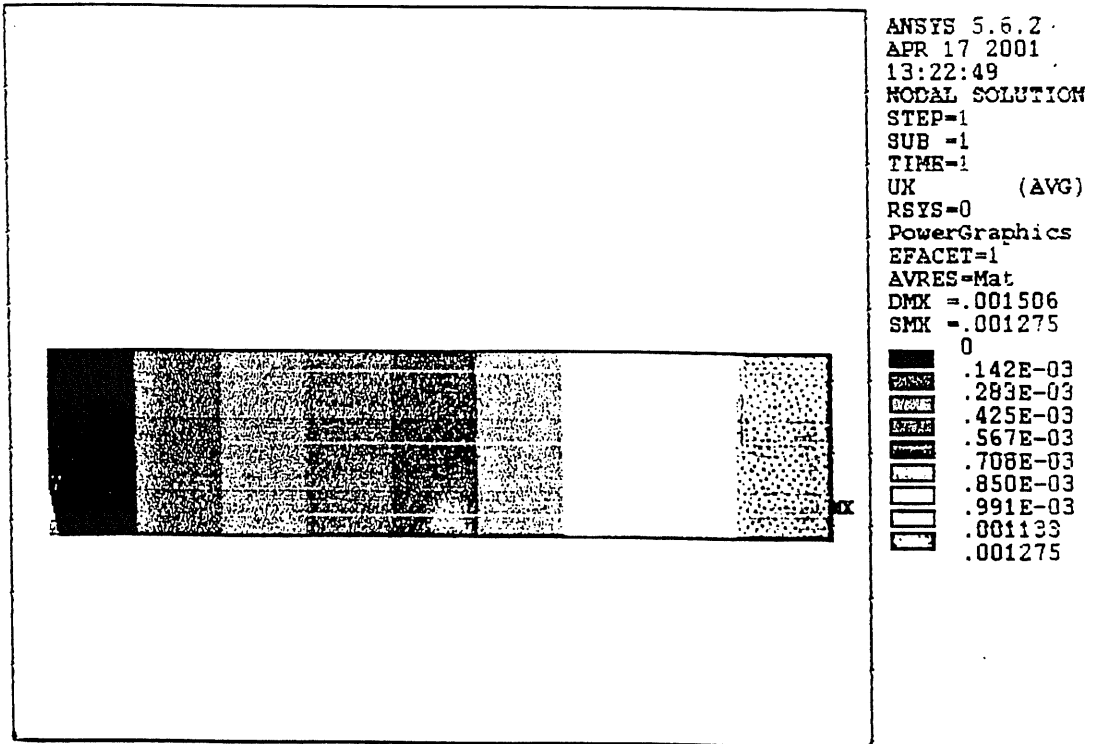


Figure 2, Contours of the displacement distribution in the x-direction

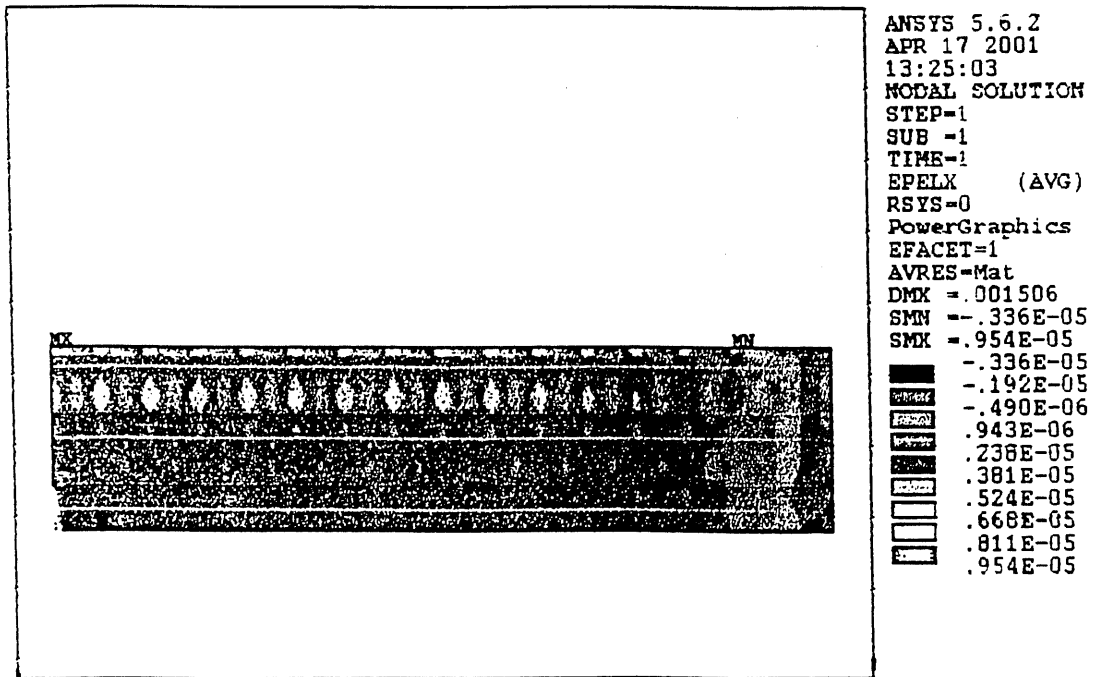
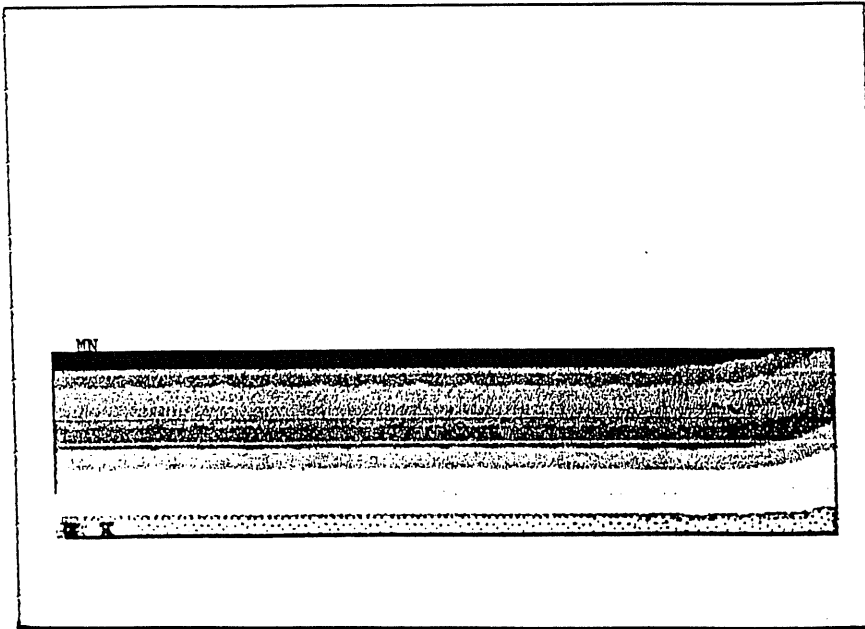


Figure 3 Distribution of mechanical strain in the x-direction

TABLE I PROPERTIES OF SANDWICH TEST SAMPLE

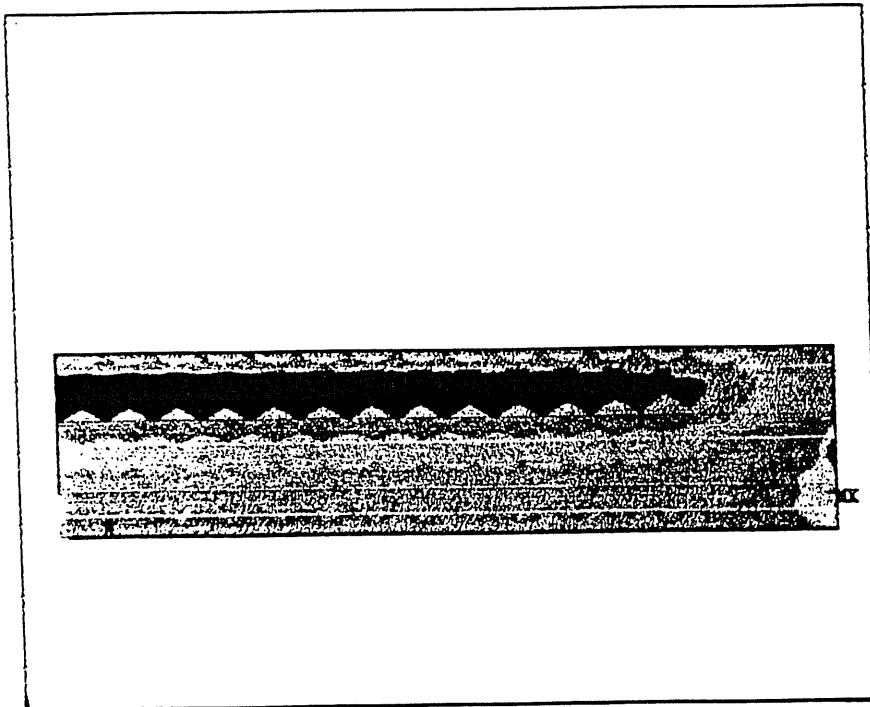
FACESHEETS	
Laminate Fiber	M60J graphite (Toray)
Laminate resin	954-2A toughened cyanate ester (Fiberite)
Dimensions	203.2 x 50 x 0.72 (mm)
Ply thickness	0.12e-3 m
Ply Axial Stiffness $E_x$	0.38e12 GPa
Ply Transverse Stiffness $E_y$	0.89e10 GPa
Ply Shear Stiffness	0.76e10 GPa
Ply Poisson's Ratio	0.2
Ply Axial CTE $\alpha_1$	-1.37 +/- -0.16 (e-6/deg C) (-170 to +170deg C)
Ply Transverse CTE $\alpha_2$	32.53 +/- 0.61 (e-6/deg C) (-170 to +170 deg C)
Laminate layup	(0/30/-30)s
A11	0.1973e9 Pa
A22	0.1901e8 Pa
A12	0.3326e8 Pa
CTE-1 or $\alpha_x^L$	-3.2189 (e-6/deg C)
CTE-2 or $\alpha_y^L$	11.7773 (e-6/deg C)
Laminate orientation	the 90 deg direction is the 203.2 mm (length) direction
ADHESIVE (estimated)	
Thickness	<0.02 mm
Adhesive stiffness	< 2.9 G Pa
Adhesive CTE	50e-6/K
CORE	
Material	Aluminum (vented)
Core thickness	50.8e-3 m
Cell Side "a"	3.1 mm
Cell diameter "d"	6.287 mm
Cell expansion angle $\theta$	55.5 degree
Core material stiffness $E_c$	6.896e10 Pa
Core material CTE	22 (e-6/degC)
Core orientation	Ribbon direction corresponds to the 0-deg direction of the facesheet, i.e., in the 50 mm sample dimension



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Figure 4 Contours of the displacement distribution in the y-direction showing some end effects



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Figure 5 Strain distribution in the y-direction

## CTE MODEL PREDICTIONS

An analytical model [14-17] was been used to predict the hygrothermal expansion of sandwich structures.) The appendix gives the basic equations. Using the parameters of Table I, values of  $\alpha_y = 19.84 \text{ e-6/K}$  in the length direction and  $\alpha_x = -3.66\text{e-6/K}$  in the transverse direction were predicted. Parametric analyses with this model indicate that the sandwich length CTE  $\alpha_y$  peaks at  $\theta = 60$  deg and that it is more sensitive to variation in  $\theta$  as the core thickness increases. As the core thickness decreases to zero, the  $\alpha_y$  value reduces to the laminate value (of  $11.777\text{e-6/K}$ ) for all values of  $\theta$  as expected. The predicted results are especially sensitive to the choice of  $\alpha_2^L$ , the transverse ply CTE.

## MODIFIED BULK CTE MODEL

Figure 6 indicates that the compressive stresses imposed by the facesheets on the core (on heating) decay through the core thickness. Linear elastic laminate plate theory requires that  $U_x(z)$  and  $U_y(z)$  should be constant for all values of  $z$ . If the core had no stiffness at all,  $U_z$  should become  $U_z(\text{max})$  at very small values of  $z$ , and correspond to the expansion of the core alone. Therefore, we postulate that the effective core stiffnesses  $Q_{ij}$  are reduced by the area under the curve  $(U_x(z) - U_x(z=0)) / (U_x(\text{max}) - U_x(z=0))$  versus  $z$  for  $0 < z < hc/2$ , essentially normalized Figure 6. If we fit Fig. 6 to a  $1 - \exp(-kz)$  type relation, then

$$\text{Fraction reduction in } Q_{ij} \text{ of core} = \frac{2}{hc} \int_{Z=0}^{Z=hc/2} 1 - e^{-kz} dz \quad (1)$$

As shown in the appendix, the  $Q_{ij}$  are directly proportional to  $E_c$ , so the calculation can be readily made by reducing  $E_c$ . With a curve fit to Fig. 6 giving  $k = 3.8/hc$ , the fraction reduction is 67 % in  $E_c$  which predicts a CTE in the length direction equal to  $15\text{e-6/K}$ . Past work on the applicability of Saint-Venant's principle [1-3] to sandwich structures suggests  $k = f(E_c/E_L, hc/h_L)$  where  $L$  refers to the laminate facesheet. However, since the facesheet in this case is anisotropic, there will be further dependence on geometry. For example, the displacement  $U_y(z)$  for  $x=0, y=25 \text{ mm}$  is an order of magnitude less than that of Figure 6.

## INTERFEROMETRIC CTE TEST

A Michelson interferometer [19] was used to measure  $\alpha_1, \alpha_2$ , (plies) and  $\alpha_y$  of the sandwich on three samples each (see Table I). The ply samples were  $177 \times 38 \times 0.85 \text{ mm}$  with fibers either longitudinally or transversely placed. In this test, lines are inscribed with a razor in the topmost ply at  $10 \text{ mm}$  from the sample ends. The mirrors whose relative motion is to be measured have knife edges and

sit in the middle of these grooves. Any longitudinal CTE variational effects are averaged out. Transverse CTE variations will promote no errors as long as the mirrors can slide in the grooves, which are less than half a ply deep.

## LOCALIZED CTE OPTICAL TECHNIQUE

An optical technique [22] was used to verify the iso-CTE in-plane contours of the facesheet predicted by the FEA model. Two stationary 2 mW, 1 mm diameter HeNe laser beams reflect diffusely at two facesheet/Ag paint line interfaces. Expansion of the sample at that spot changes the integrated intensities of the reflected light, and these are measured with two identical optical systems. Calibration of the photodiode output voltage is carried out in situ in the vacuum oven with an LVDT connected to a motorized remote micropositioner system. Samples were dried prior to all measurements. The average of three thermocouples on both sides of the sample is used for temperature. Figure 7 shows the microstrain vs. temperature data for two center tests and two corner tests. A slight reduction in slope (CTE) is noted at the corner tests. The scatter (correlation coefficients) in the data reflect ongoing technique development.

## RESULTS

Table II summarizes the predicted and measured results. The optical methods suggest a slightly larger decrease in CTE at the edge/corner than is predicted by the FEA analysis. When the decreasing effect of core stiffness with increasing core thickness is taken into account there is good agreement with all theory and experiment.

## CONCLUSIONS

Both the FEA and laminate models depend on accurate inputs of laminate properties, in particular the transverse ply CTE and the ply shear stiffness. It is important to compare CTE data over the same temperature range, as for these and most classes of materials there is a slight curvature indicating higher CTE values at higher temperatures. The laminate model overpredicts the measured and FEA model predicted CTEs because it assumes that the effective core stiffness is constant throughout its thickness. Evidence of core bulging suggests that the stresses imposed on the core by the facesheets are dissipated as the core gets thicker and this reduces the predicted extension of the facesheet. The interferometric technique can not account for transverse CTE variations smaller than the width of the mirror system used. The optical reflectance technique is subject to some scatter, possibly due to out-of plane (z-direction) expansion. The FEA model indicates periodic fluctuations in localized strain on the facesheets caused by honeycomb geometry. This may also help to account for CTE variations depending on the sensor location.



TABLE II SUMMARY OF PREDICTED AND MEASURED CTE RESULTS IN PPM/K

	Central $\alpha_y$	Edge $\alpha_y$
<b>FEA Prediction (-170 to +170 K)</b>		
2mm from sample end	14.67	14.54
12 mm from sample end	14.77	
50 mm from sample end	14.83	14.84
<b>CTE Model Prediction</b>		
Modified for stress decay	19.84	15
<b>Interferometric Measurement (3 samples)</b>		
12 mm from sample end	12.78 +/- 0.095	(-123 to 84 K)
8 mm from sample end, heat	13.70 +/- 0.13	(24 to 84 K)
cool	12.48 +/- 0.23	(24 to 84 K)
<b>Optical Reflectance Technique (24 to 80 K)</b>		
50 mm from end	13.877 +/- 0.16	
2mm from end		11.925 +/- 0.042

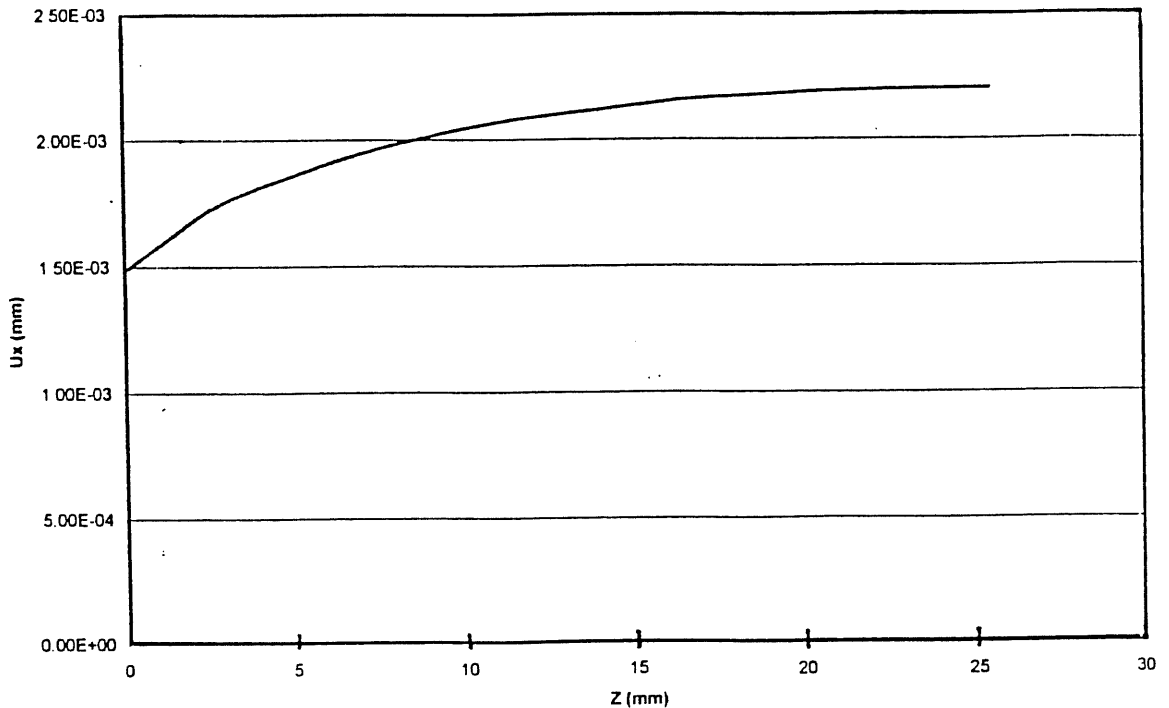


Figure 6 Aluminum honeycomb displacement as  $f(z)$  at  $x = 101.6\text{mm}$  and  $y = 0$

## RECOMMENDATIONS

The analyses and measurements described in this paper need to be extended to other sandwich structures to further validate the models and techniques described. In particular, measurements of transverse CTE are needed with the optical reflectance method to check the FEA model predictions of significant transverse CTE variability. Tests are also recommended for sandwich structure utilizing relatively thinner and/or less stiff facesheets, such as Kevlar, Nomex, etc. here edge/end effects would be greater and more attention is needed to positioning of grips, sensors, etc.

## ACKNOWLEDGMENTS

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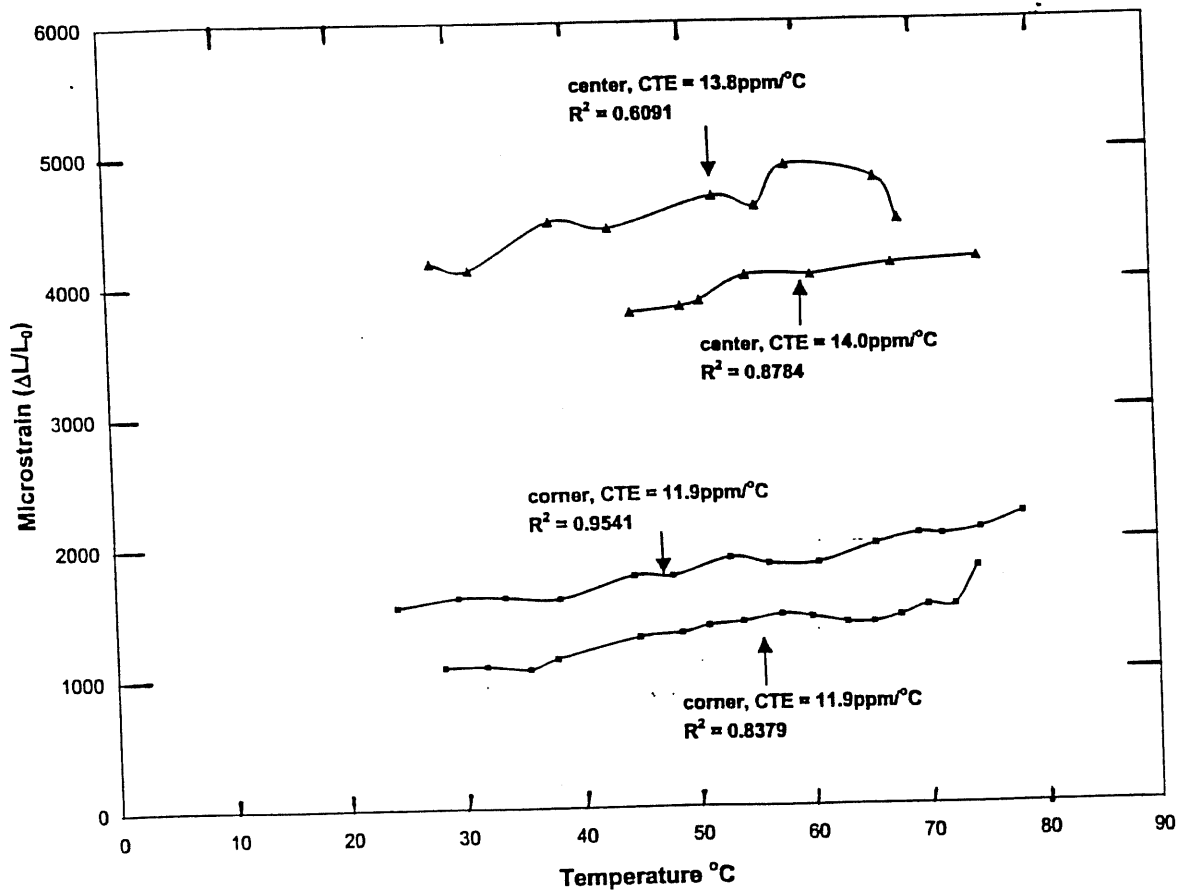


Figure 7 Microstrain vs. Temperature for two heating runs at the sample center line (10 mm from end) and two runs at the sample corner (about 3 mm from two edges)

## APPENDIX

The thermal expansion coefficients of a sandwich structure are given in terms of general linear elastic laminated plate theory as;

$$\alpha_x = [A_{22}N_x - A_{12}N_y] / [A_{11}A_{22} - A_{12}^2] = \epsilon_x / \Delta T \quad (A1)$$

$$\alpha_y = [A_{11}N_y - A_{12}N_x] / [A_{11}A_{22} - A_{12}^2] = \epsilon_y / \Delta T \quad (A2)$$

x,y,z are the laminate or sandwich coordinates; 1,2,3 are the ply coordinates. The stiffness A and thermal force resultants N terms are combinations of laminate and core stiffnesses and their respective thicknesses. The core stiffnesses (Qxx, etc) are given by Ref.16,17, and assume the cell walls react only on their physical plane. The laminate A stiffnesses are readily calculated by a standard plate laminate code. Since there are two face sheets for one thickness (hc) of the core, only half the core is used. The designations, c and L refer to the core and laminate facesheet, resp. Ec is the modulus of the core material,  $\theta$  the core expansion angle, "a" the core cell side, "t" the core material foil thickness, and  $\alpha$ 's the respective CTE values. The x direction also refers to the ribbon direction (which is then aligned with the 0-deg direction of the facesheet). See text for modifications to account for reduction of effective Q's with increasing hc. For midplane symmetrical laminates:

$$A_{11} = A_{11}^L + Q_{xx} * hc/2 \quad (A3)$$

$$A_{22} = A_{22}^L + Q_{yy} * hc/2 \quad (A4)$$

$$A_{12} = A_{12}^L + Q_{xy} * hc/2 \quad (A5)$$

$$N_x = A_{11}^L \alpha_x^L + A_{12}^L \alpha_y^L + Q_{xx} * hc/2 * \alpha_x^c + Q_{xy} * hc/2 * \alpha_y^c \quad (A6)$$

$$N_y = A_{12}^L \alpha_x^L + A_{22}^L \alpha_y^L + Q_{xy} * hc/2 * \alpha_x^c + Q_{yy} * hc/2 * \alpha_y^c \quad (A7)$$

$$Q_{xx} = Ec / [ \gamma H1(\theta) H3(\theta) ] \quad (A8)$$

$$Q_{yy} = Ec / [ \gamma H2(\theta) H3(\theta) ] \quad (A9)$$

$$Q_{xy} = Q_{yx} = [H4(\theta) Ec] / [ \gamma H1(\theta) H2(\theta) H3(\theta) ] \quad (A10)$$

$$H1(\theta) = \sin^3 \theta / [ 1 + \cos \theta ] \quad (A11)$$

$$H2(\theta) = [(1 + \cos \theta) \cos^2 \theta] / \sin \theta \quad (A12)$$

$$H3(\theta) = 2 + [1 + \cos^2 \theta] / \sin^2 \theta + [\sin^2 \theta / \cos^2 \theta] \quad (A13)$$

$$H4(\theta) = \sin \theta \cos \theta \quad (A14)$$

$$\gamma = a/t \quad (A15)$$