NATIONAL RESEARCH COUNCIL
CANADA

DIVISION OF BUILDING RESEARCH

UNBALANCE ERRORS IN GUARDED HOT PLATE MEASUREMENTS

BY

WILLIAM WOODSIDE AND A. G. WILSON

REPRINT FROM AMERICAN SOCIETY FOR TESTING MATERIALS SYMPOSIUM ON
THERMAL CONDUCTIVITY MEASUREMENTS AND APPLICATIONS OF
THERMAL INSULATIONS. A. S. T. M. SPECIAL TECHNICAL
PUBLICATION NO. 217. 1957. P. 32-46.

RESEARCH PAPER NO. 48

OF THE

DIVISION OF BUILDING RESEARCH

OTTAWA

FEBRUARY 1958

NRC 4450

PRICE 25 CENTS

This publication is being distributed by the Division of Building Research of the National Research Council as a contribution towards better building in Canada. It should not be reproduced in whole or in part, without permission of the original publisher. The Division would be glad to be of assistance in obtaining such permission.

Publications of the Division of Building Research may be obtained by mailing the appropriate remittance, (a Bank, Express, or Post Office Money Order or a cheque made payable at par in Ottawa, to the Receiver General of Canada, credit National Research Council) to the National Research Council, Ottawa. Stamps are not acceptable.

A coupon system has been introduced to make payments for publications relatively simple. Coupons are available in denominations of 5, 25, and 50 cents, and may be obtained by making a remittance as indicated above. These coupons may be used for the purchase of all National Research Council publications including specifications of the Canadian Government Specifications Board.

Authorized Reprint from the Copyrighted
Symposium on Thermal Conductivity Measurements and
Applications of Thermal Insulations
Special Technical Publication No. 217
Published by the
AMERICAN SOCIETY FOR TESTING MATERIALS
1957

UNBALANCE ERRORS IN GUARDED HOT PLATE MEASUREMENTS

BY W. WOODSIDE[1] AND A. G. WILSON[1]

SYNOPSIS

The errors in thermal conductivity measurement in three different designs of guarded hot plate caused by small temperature differences (unbalances) between test area and guard ring are described. The dependence of these errors upon: (*a*) the magnitude and direction of the unbalance, (*b*) the size and design of heater plate, (*c*) the conductivity and thickness of the specimens tested, and (*d*) the temperature difference between hot and cold plates is investigated. The percentage error in measured conductivity is shown to increase as the conductivity of the specimens tested decreases. Errors as high as 6 per cent were obtained even when the ASTM balance requirement was satisfied.

There are two constants for any heater plate which determine its error sensitivity to unbalance. If these constants are known, the maximum tolerable unbalance required to achieve any desired accuracy in measured conductivity may be calculated. The experimental determination of these constants, involving a minimum of two tests on each of two sets of specimens of different conductivity, is described.

The guarded hot plate is the most widely used apparatus for the precise determination of the thermal conductivity of insulating and other building materials. It has been standardized by the ASTM.[2] During a test, the temperature of the guard ring is held as closely as possible to the test area temperature by either manual adjustment of the guard ring heating current, or some system of automatic control. The ASTM method of test requires that during the 5-hr equilibrium period the temperature difference between the test area and guard ring should not exceed 0.75 per cent of the temperature difference between hot and cold plates. One per cent is the limit specified by RILEM[3] (1).[4]

Gilbo (2), working with an 8-by 8-in. hot plate and testing 1-in. thick cork specimens with a deep groove cut into the faces of the specimens so that the heater contact area and guard contact area were separated found that a temperature difference or unbalance between the test area and guard ring of 0.2 F caused an error in the conductivity of

---

[1] Research Officer and Head respectively, Building Services Section, Division of Building Research, National Research Council of Canada, Ottawa, Canada.

[2] Method of Test for Thermal Conductivity of Materials by Means of the Guarded Hot Plate (C 177 – 45), 1955 Book of ASTM Standards, Part 3, p. 1084.

[3] Réunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions (RILEM). An international materials testing organization.

[4] The boldface numbers in parentheses refer to the list of references appended to this paper, see p. 46.

N. R. C. 4450

3 per cent. The temperature difference between hot and cold plates was not stated. He attributed part of the disagreement among results of several laboratories to lack of temperature balance between test area and guard ring.

Roux *et al* (3) investigated the errors due to unbalances of the order of 0.03 F in their 12-in. hot plate, using four different materials. They concluded that such an unbalance would produce errors of 0.1 per cent for a cement plaster (k = 7.06), 0.16 per cent for yellowwood (k = 1.00), 0.27 per cent for masonite insulation board (k = 0.40), and 0.52 per cent for cork (k = 0.30 Btu in. per hr sq ft deg Fahr).

TABLE I.—DIMENSIONS OF HEATER PLATES.

Heater Plate	Over-all Size, in.	Size of Test Area, in.	Width of Guard Ring, in.
A.......	8 by 8	4 by 4	2
B.......	8 by 8	4 by 4	2
C.......	18 by 18	12 by 12	3

More recently Pascal (4) also noted that the error due to unbalance depended upon the conductivity of the specimen tested, being greater the lower the conductivity. The test and guard areas of his 50-by 50-cm hot plate were separated by a 3-mm thickness of araldite insulation. By calculating the lateral heat flow directly through this araldite separator for a given unbalance, he obtained the maximum allowable unbalance for any desired precision in the conductivity determination and for any power input to the test area. This, however, neglected the lateral flow through the specimens themselves. Pascal recognized that a 3-mm air gap would have a thermal resistance to lateral flow six times as great as his 3-mm araldite gap, but retained the araldite for reasons of surface continuity and mechanical strength of the plate. However, as a result of this low lateral thermal resistance, errors caused by a given unbalance are large compared with those for a similar plate with an air gap. For example, Pascal, in testing cork 10 cm thick, found an error of 35 per cent in conductivity due to an unbalance of 1 per cent of the temperature difference between hot and cold plates.

This present paper deals with the unbalance errors of three different designs of guarded hot plates and the factors that influence these errors. It is shown how errors due to unbalance may be calculated, knowing two constants for the hot plate and the conditions under which the test is performed. A method for the simple determination of these constants is described.

APPARATUS

The three heater plates tested for sensitivity to thermal unbalance between the test area and guard ring were supported vertically. The dimensions of the plates are shown in Table I. Plate A is patterned after the U. S. National Bureau of Standards (NBS) modified guarded hot plate apparatus originally described by van Dusen (5) and has a $\frac{1}{16}$-in. gap separating test and guard areas, with $\frac{1}{16}$-in. thick copper surface plates. Plate B is similar to one designed and used successfully at the University of Saskatchewan and is very much simpler and less expensive to construct than the NBS design. It has a $\frac{1}{8}$-in. gap between test and guard areas and $\frac{1}{4}$-in. thick aluminum surface plates. Both heater plates A and B were used in conjunction with cold plates of NBS design. Plate C has been used by the National Research Laboratories for several years and has been described by Niven (6). It has $\frac{1}{8}$-in. thick copper surface plates. The gap width of plate C is not uniform, being greater at the top of the test area than at the other three sides and varying between 0.121 and 0.082 in. The average gap width is 0.0935 in. The details at the gaps of the three heater plates are shown in Fig. 1.

The temperature difference between test area and guard ring is measured differently in each of the heater plates, as shown in Fig. 2. In plate A the unbalance is measured by four differential thermocouples in series, to ±0.0012 F. In plate B the unbalance is measured by two sets of two differential thermocouples in series, to ±0.0024 F. In plate C, the unbalance is taken as the difference between the temperature averages of the four guard ring and four test area thermocouples and is measurable to ±0.005 F.

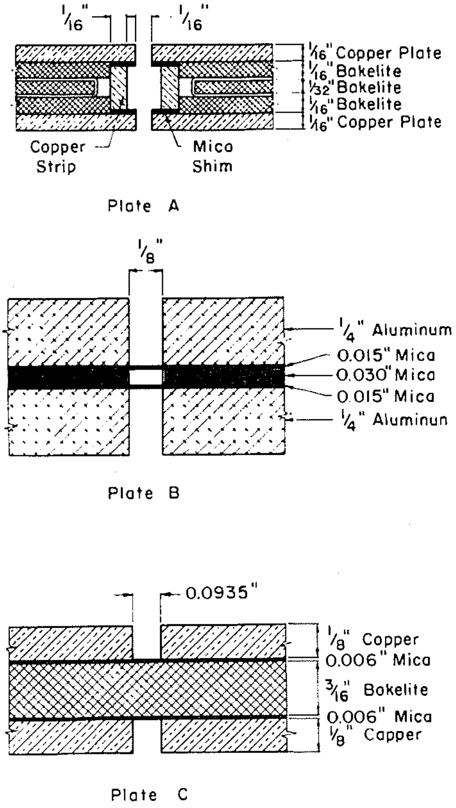

FIG. 1.—Details at Gaps of Heater Plates.

In all the test work to be described, the guard-ring heater current was controlled manually to produce thermal balance or any desired unbalance between test and guard areas.

The source of direct current supplied to the heater plates is a motor-generator set with an output of 110 v. This is regulated and controlled by d-c voltage regulator-controller assemblies based on a circuit designed by Dauphinee and Woods (7) which reduce voltage fluctuations by a factor of 100 and permit continuous variation of the regulated output fed to the heaters from 0 to 70 v.

The cold plates are supplied with a water-ethylene glycol mixture from a 90-gal reservoir containing heating and cooling units and automatically maintained by a mercury thermostat at any desired temperature to better than ±0.1 F.

In all tests the edges of the specimens were exposed to the ambient air in the laboratory. The laboratory was conditioned to 73 ± 1 F and to relative humidities below 15 per cent.

EXPERIMENTAL PROCEDURE

Materials tested in the three sets of hot plate apparatus included the following in order of increasing conductivity: silica aerogel, rock wool batt insulation, expanded polystyrene, cellular poly vinyl-chloride, fiberboard, plywood, cellular concrete, Lucite, sand, and rubber, so that a range of thermal conductivity from 0.15 to 2.30 Btu in. per hr sq ft deg Fahr was covered. Specimens of the materials that are moisture sensitive were conditioned to constant weight in a ventilated oven at 140 F prior to each test. All tests were performed at a mean temperature of 75 F.

The specimens were installed in the apparatus and a test performed to determine their apparent conductivity with a temperature unbalance between

Note: Similar Pairs of Differential Thermocouples at Top and Bottom Edges of Gap in other Surface Plate of Plate B.

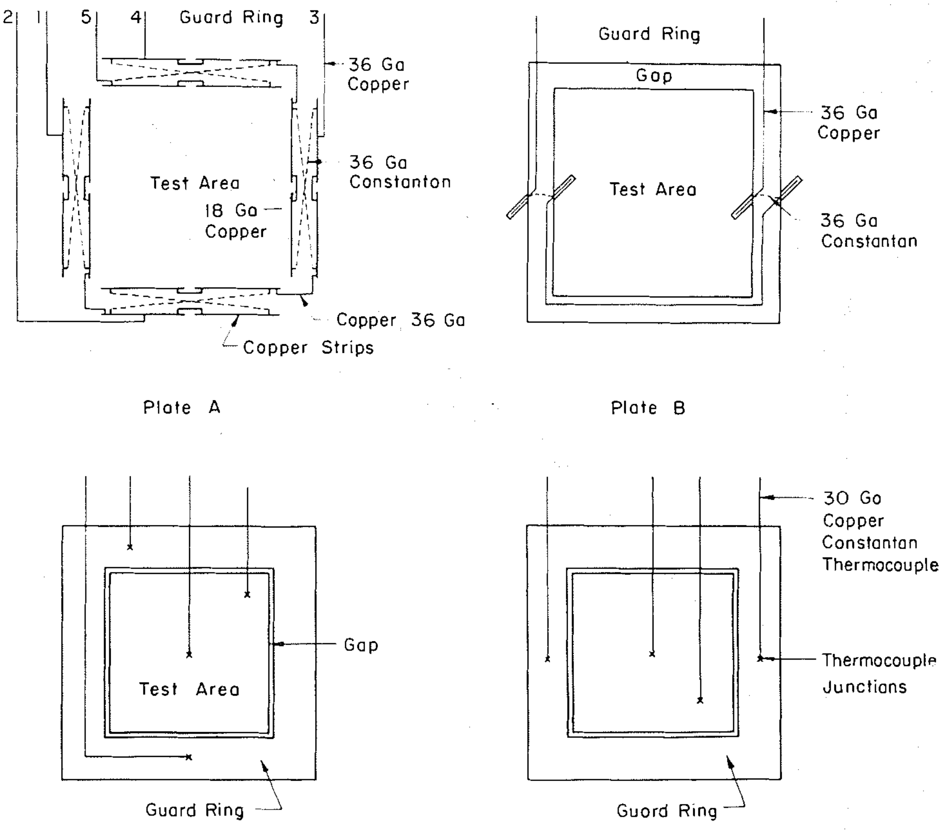

Plate A

Plate B

Plate C

FIG. 2.—Thermocouples Used to Measure Temperature Unbalance.

test area and guard ring. When the temperature difference between hot and cold plates, the temperature difference between test and guard areas, and the power input to the test area had remained constant for a 5-hr period, the apparent thermal conductivity, k_{exp}, was calculated. This was repeated several times, each time with a different temperature unbalance between test and guard areas, but with all other conditions unchanged. The unbalances varied between +3.0 and −3.0 F, taking the positive sign to indicate that the test area was at a higher temperature than the guard ring. Thus for each value of the unbalance $\Delta\theta$ a value of the apparent conductivity k_{exp} was obtained. Since all other conditions such as mean temperature, density, and moisture content were held constant in each test series, the variation of k_{exp} was caused only by the variation of $\Delta\theta$.

This procedure was followed with the

three different sets of apparatus and for each material tested.

In addition, several tests were performed, in which the hot plate to cold plate temperature difference θ was varied as well as the unbalance.

To determine the effect of specimen thickness upon unbalance errors, Lucite was tested with plate A at two different thicknesses—$\frac{3}{4}$ in. and 2 in. The ASTM limits specimen thicknesses to 1 in. for a heater plate of dimensions equal to those of plate A. However, the 2-in. Lucite specimens were being tested as part of another test series designed to determine errors due to edge heat loss to the ambient air, and it was therefore convenient to use them for the unbalance tests.

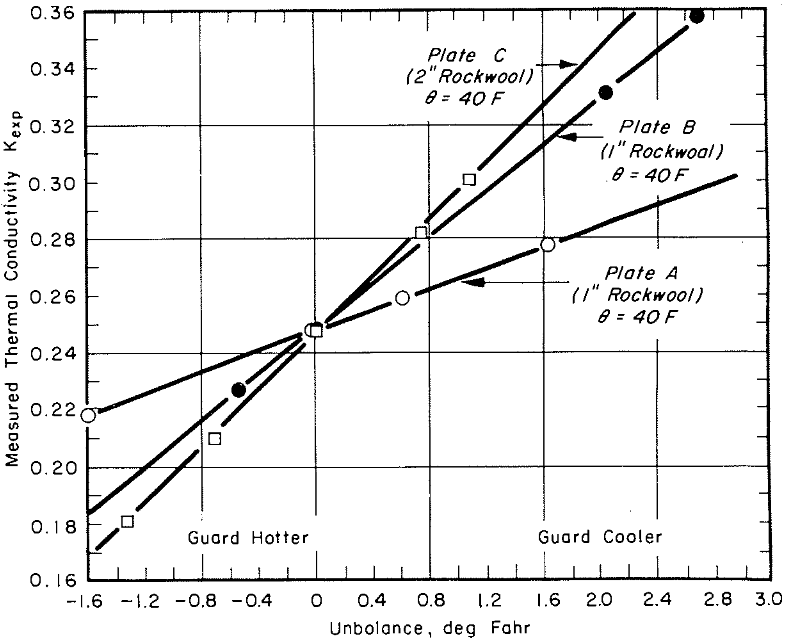

FIG. 3.—Effect of Temperature Unbalance on Measured Conductivity of Rock Wool for All Plates.

TEST RESULTS AND ANALYSIS

In Fig. 3, k_{exp} is plotted against $\Delta\theta$ for rock wool batt insulation tested with the three different heater plates. The straight-line relationship between k_{exp} and $\Delta\theta$ shown in this figure is typical of the results obtained with all plates and all specimens tested. The true value of the thermal conductivity k is taken to be the value of k_{exp} when $\Delta\theta = 0$ and can be read directly from the graph. As can be seen, excellent agreement between the three plates was obtained.

When the guard ring is at a higher temperature than the test area, that is, for $\Delta\theta < 0$, the test area receives heat from the guard section, and hence a smaller power input Q_{exp} to the test area is required to maintain a given temperature gradient through the specimen. This means that for $\Delta\theta < 0$ the measured thermal conductivity k_{exp} is less than the true conductivity k, since for $\Delta\theta \neq 0$, $k_{exp} = Q_{exp} L/A\theta$;[5] and for $\Delta\theta = 0$,

---

[5] Where:

Q = total heat input to test area, Btu per hr, when $\Delta\theta = 0$,
Q_{exp} = total heat input to test area, Btu per hr, when $\Delta\theta \neq 0$,
A = test area, sq ft (both faces of heater plate),
L = specimen thickness, in.

$k = QL/A\theta$. The reverse holds for $\Delta\theta > 0$, that is, when the guard is cooler than the test area, $k_{\text{exp}} > k$.

In Fig. 4, $\Delta k \times 100/k$, the percentage error in conductivity caused by an unbalance $\Delta\theta$ is plotted against $\Delta\theta \times 100/\theta$, the unbalance expressed as a percentage of the temperature difference between hot and cold plates, for plate A and ½-in. fiberboard. The temperature difference θ was varied between 39 and 58 F in these tests. As can be seen from this figure, the error is directly proportional to $\Delta\theta \times 100/\theta$ over the range covered (approximately −3.0 to +6.0 per cent). To determine if $\Delta k/k$ is proportional to $\Delta\theta$ for larger unbalances, a single test was performed on the same specimen with plate A, the test being conducted with zero guard ring power input. This test gave an error of 76 per cent in conductivity at $\Delta\theta \times 100/\theta = +60$. By extrapolating the line in Fig. 4 to this value of $\Delta\theta \times 100/\theta$, an error of 69 per cent is found. Thus the error remained approximately proportional to $\Delta\theta$ even for the highest positive unbalance attainable in this test. In Figs. 5, 6, and 7, $\Delta k \times 100/k$ is plotted against $\Delta\theta \times 100/\theta$ for plates A, B, and C, respectively, for all specimens tested.

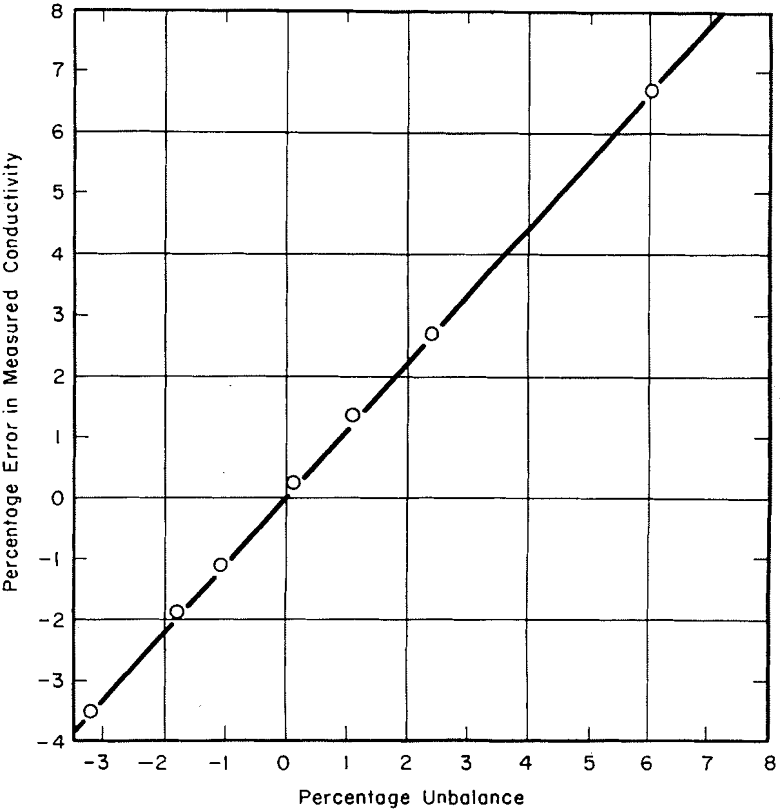

FIG. 4.—Percentage Error in Measured Conductivity, Determined at Several Temperature Differences Between Hot and Cold Plates, *versus* the Unbalance Expressed as a Percentage of the Temperature Difference.

When a temperature difference exists between the central test area and the guard ring, not only will heat be transferred directly across the gap, but also heat, which originated in the test area

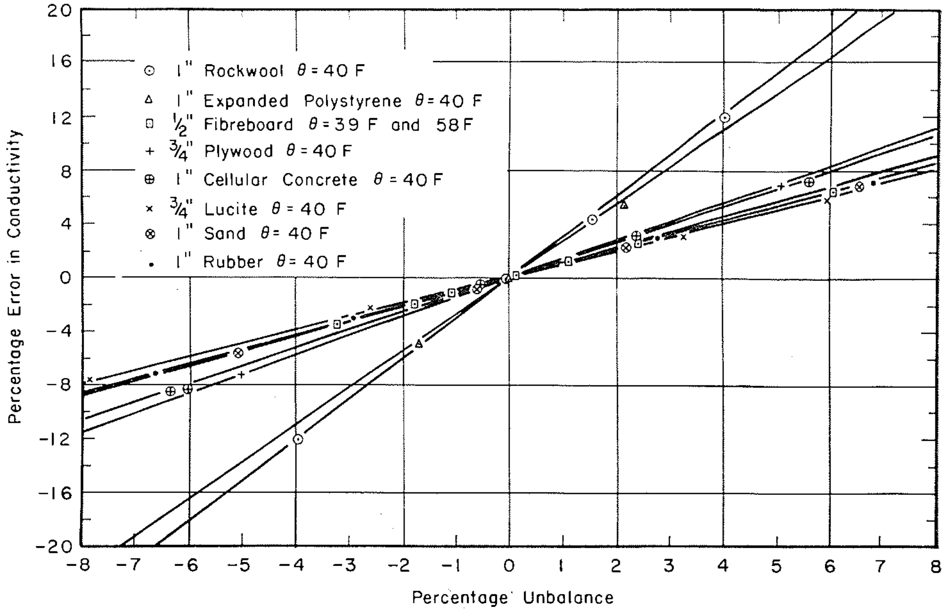

FIG. 5.—Percentage Error in Measured Conductivity *versus* Percentage Unbalance for Materials Tested with Plate A.

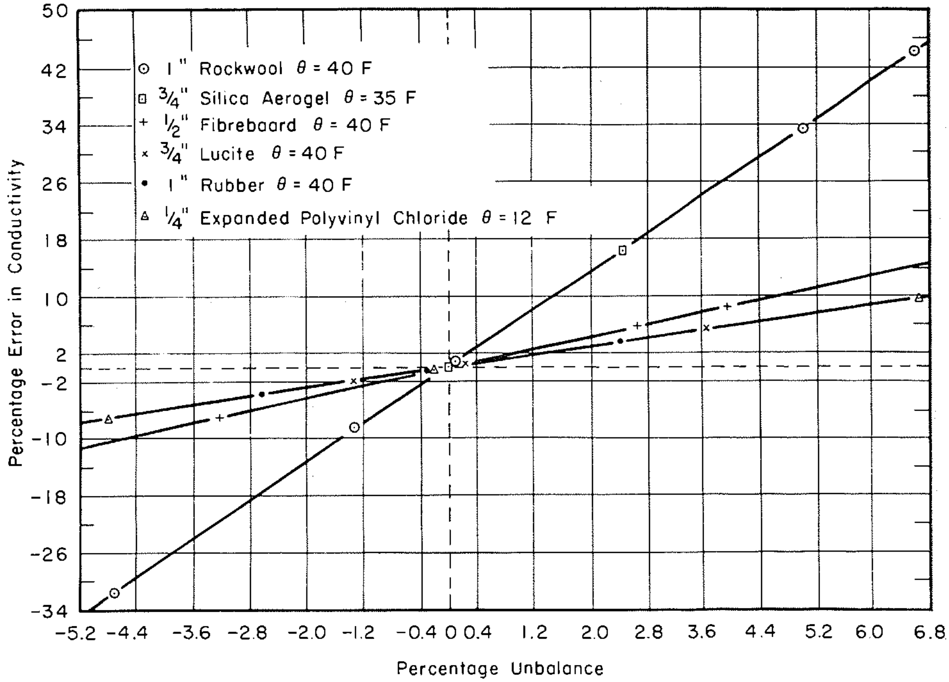

FIG. 6.—Percentage Error in Measured Conductivity *versus* Percentage Unbalance for Materials Tested with Plate B.

of the heater plate, will be conducted through the specimens toward that part of the cold plate which is opposite the guard ring. This occurs when the unbalance is positive; the reverse takes place with a negative unbalance. This means that the heat flow and isothermal surfaces in the specimen will be distorted. It is the sum of these two heat flows, that is, the heat flow directly across the gap and that which is displaced out of or into the central test area section of the samples, which will be termed "error heat flow" and which results in an error in the measured conductivity. The method for calculating this error heat flow from the test results is described below.

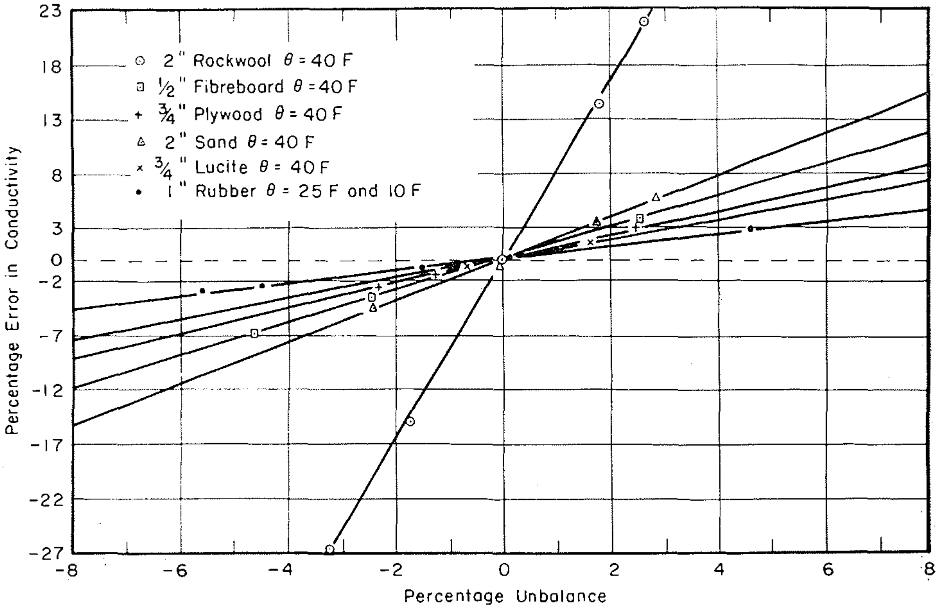

FIG. 7.—Percentage Error in Measured Conductivity *versus* Percentage Unbalance for Material Tested with Plate C.

Let the normal heat flow from the test area under balanced conditions be Q Btu per hr. Suppose that the guard ring is $\Delta\theta$ F cooler than the test area. Let the error heat flow from test area to guard ring area be denoted by q Btu per hr. The total heat flow out of the test area will now be $(Q + q)$.

When $\Delta\theta = 0$,

$$k = QL/A\theta \qquad (1)$$

In the unbalanced condition, the experimentally determined conductivity is:

$$k_{\text{exp}} = (Q + q)\, L/A\theta \qquad (2)$$

$$\therefore \Delta k = k_{\text{exp}} - k = qL/A\theta$$

$$\therefore q = Q\,\Delta k/k \qquad (3)$$

or using Eq 1

$$q = A\,\theta\,\Delta k/L \qquad (4)$$

From the plot of k_{exp} *versus* $\Delta\theta$, the error Δk in conductivity for any unbalance $\Delta\theta$ may be obtained directly, and hence using Eq 4, q, the total error heat flow for any unbalance $\Delta\theta$, may be calculated.

Using Eq 4, q for a 1 F unbalance has been calculated for each specimen tested in each of the three plates. In Fig. 8 these values are plotted against conductivity, and it is evident that q increases approximately linearly with k. The scatter in the experimental points for plate C may be caused by the less accurate method of measuring unbalances in this heater plate. In plate C, the thermocouples which detect the unbalance are not located adjacent to the gap but some distance from it (Fig. 2), and hence when there is a temperature gradient across the guard-ring surface the unbalance measured is not the temperature difference directly across the gap.

When $k = 0$, that is, when perfectly insulating specimens are placed on each side of the heater plate, $q = q_0$. Therefore q_0 is the lateral heat transfer directly across the gap when there is a 1 F temperature unbalance between test and guard areas. Thus q_0 is a plate constant which depends only on the size and design of the heater plate, and is one of the two characteristics of any guarded hot plate which determine its sensitivity to unbalance. The other is the slope of the q *versus* k graph which is denoted by c. This second plate constant depends only upon the size of the plate and the gap width. Comparing the graphs of q *versus* k for plates A and B, which are both of the same size but have different gap widths, it is seen that the larger the width of the gap separating test and guard areas, the smaller the value of c. The ASTM method specifies $\frac{1}{8}$ in. as the maximum gap width. By making the gap width larger, errors due to unbalance will be smaller, but the size of the test

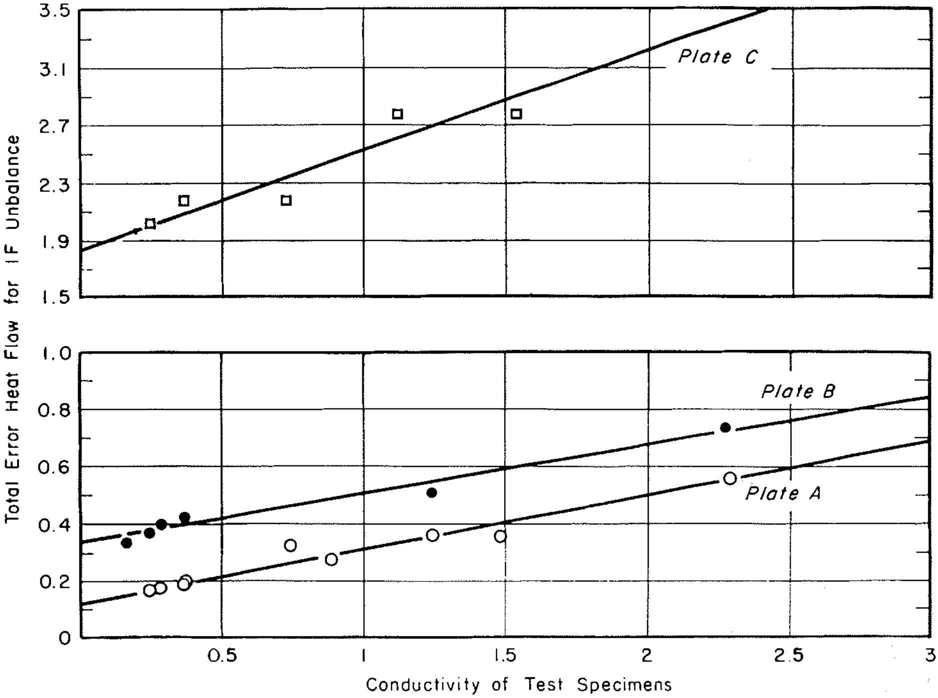

FIG. 8.—Dependence of Total Error Heat Flow for a 1 F Unbalance upon Conductivity.

TABLE II.—VALUES OF q_0 AND c FOR THE THREE PLATES.
(Btu per hr deg Fahr unbalance)

Heater Plate	q_0	c
A	0.120	0.192
B	0.335	0.170
C	1.840	0.680

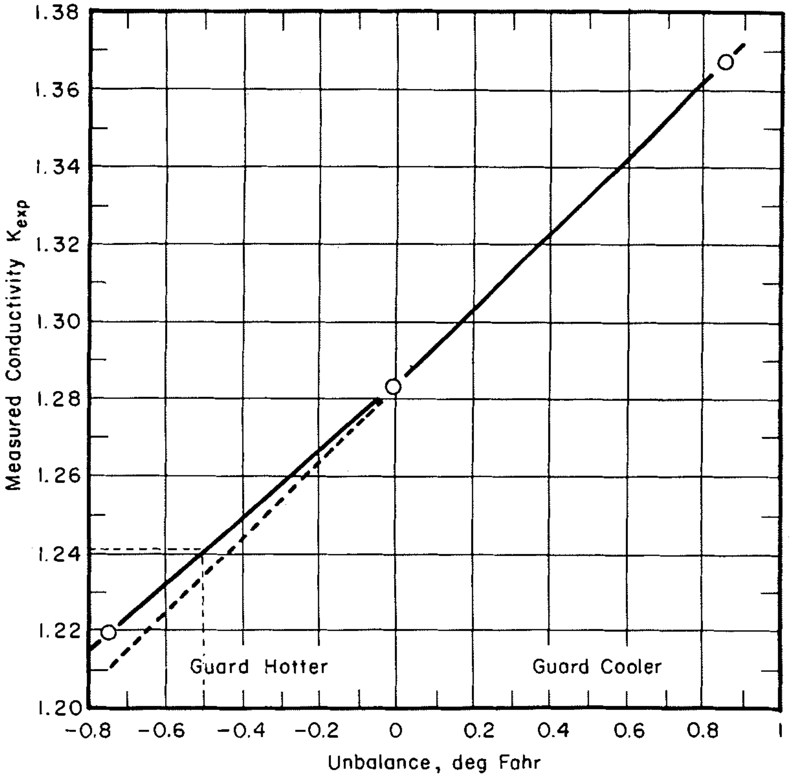

FIG. 9.—Measured Conductivity of 2 in. Lucite Plotted Against Temperature Unbalance Between Test and Guard Areas for Plate A.

area becomes more indefinite, and the distortion of the heat flow lines out of the test area, even under balanced conditions, becomes greater. The maximum specified gap width, however, should not be independent of the linear dimension of the test area. Specifying a maximum value for the ratio of gap width to linear dimension of test area would appear to be more logical.

The dependence of q upon k can be represented by:

$$q = q_0 + c\,k \qquad (5)$$

If one wishes to attach a physical meaning to c, it may be defined as that portion of the error heat flow which flows through a specimen of unit conductivity caused by an unbalance of 1 F. Equation 5 shows the breakdown of the total error heat flow into its two component parts: (*a*) the heat transfer directly across the gap and (*b*) the error flow through the specimens themselves. For materials of low conductivity, q_0 is the more important factor. Table II shows the values of q_0 and c for the three plates.

It can now be seen why, when plotting $\Delta k/k$ *versus* $\Delta\theta/\theta$ (Figs. 5, 6, and 7) different lines are obtained for materials of different conductivity. From Eq 5 the total error heat flow for an unbalance $\Delta\theta$ F is:

$$q' = (q_0 + ck)\Delta\theta.$$

$$\frac{\Delta k}{k} = \frac{q'}{Q} = \frac{(q_0 + ck)\Delta\theta}{kA\theta/L},$$

or

$$\frac{\Delta k}{k} = \frac{L}{A}\frac{\Delta\theta}{\theta}(q_0/k + c) \qquad (6)$$

Therefore for a given unbalance $\Delta\theta$, the error $\Delta k/k$ increases as the conductivity of the material decreases if the other factors L and θ remain unchanged. Thus the largest unbalance errors are obtained with low conductivity specimens, as observed by Roux *et al* (3). As k increases, q_0/k becomes smaller in comparison to c, and hence for high conductivity specimens the error for a given unbalance is less dependent upon k, as can be seen from Figs. 5, 6, and 7.

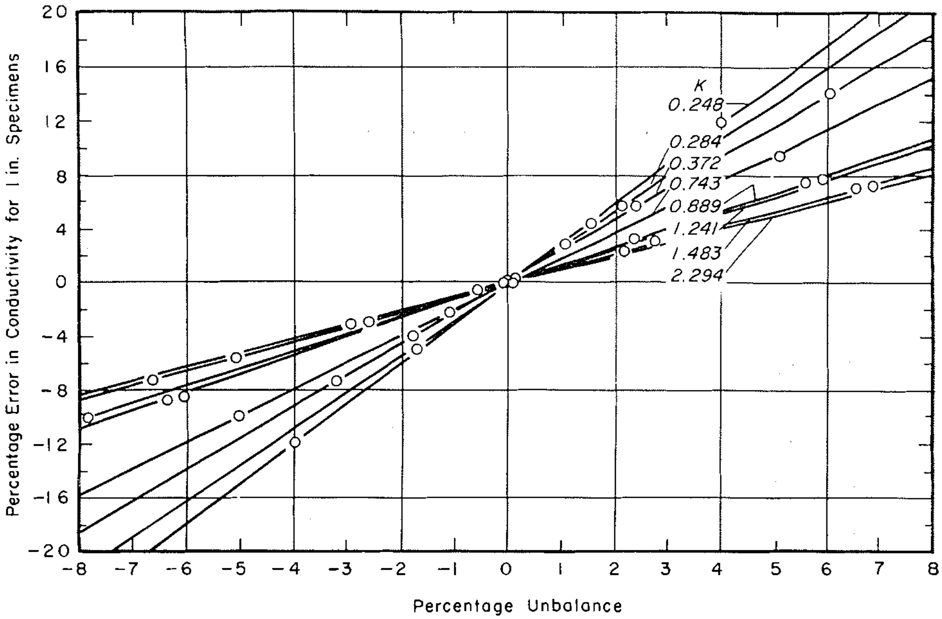

FIG. 10.—Relationship Between Percentage Error in Measured Conductivity and Percentage Unbalance, Based on Unit Thickness of Test Specimens, Showing the Effect of Conductivity upon the error.

Equation 6 predicts that the percentage error in measured conductivity is proportional to the specimen thickness tested. To verify this, Lucite was tested in plate A at two thicknesses—0.75 in. and 2.0 in.—and at different degrees of unbalance. When

$$\Delta\theta = -0.5, \qquad \Delta k = 0.016 \text{ for } L = \tfrac{3}{4} \text{ in.}$$

From Fig. 9, which is a plot of k_{exp} *versus* $\Delta\theta$ for the 2-in. specimen of Lucite, when $\Delta\theta = -0.5$ F, $\Delta k = 0.042$ (for $L = 2$ in.). Since $0.016 \times 2.0 \div 0.75 = 0.043$, the test verifies that the error in conductivity due to unbalance is directly proportional to the specimen thickness.

In Fig. 9, the value of k at $\Delta\theta = 0$ (1.28) is greater than the value obtained with the $\frac{3}{4}$-in. specimen (1.24), due to the greater heat loss from the specimen edges in the case of the thicker specimen. Also it is seen that larger errors result when the guard ring is cooler than the test area than when it is warmer by the same amount. The reason for this asymmetry is the edge heat loss which acts in the same direction as a positive unbalance and which can be counteracted by a negative unbalance. From Fig. 9, it is apparent that a −0.5 F unbalance would exactly nullify the effect of the edge heat loss. This asymmetry effect was not observed when testing thinner specimens of thickness of 1 in. or less in plate A, since there the edge heat loss effect was not "felt" in the test area. This effect could therefore be used to determine if specimens of too great a thickness are being tested in any given heater plate. If no asymmetry is apparent in the unbalance results, the edge heat loss effect is negligible.

In Figs. 5, 6, and 7, the slopes of the lines of percentage error in conductivity plotted against percentage unbalance do not decrease with increasing conductivity. For example, in Fig. 5 the line for fiberboard (k = 0.372) has a smaller slope than the lines for plywood and cellular concrete which have higher conductivities (0.743 and 0.890 respectively). This is because the specimens were all of different thicknesses. The results can be reduced to a common thickness, say 1 in., by multiplying $\Delta k/k$ by $1/L$ before plotting against $\Delta\theta/\theta$. This has been done for plate A in Fig. 10, where the slopes of the lines decrease as k increases.

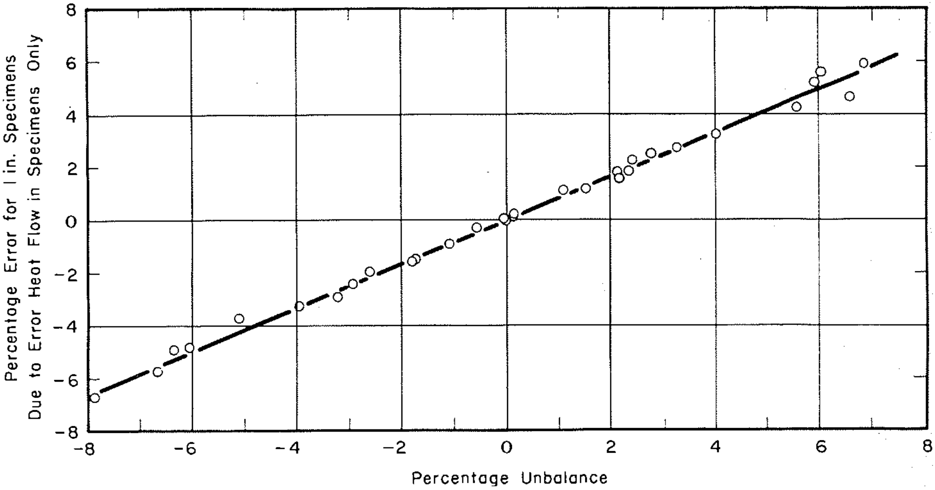

FIG. 11.—Percentage Error in Conductivity for Specimens of Unit Thickness Due to Error Heat Flow in the specimens only, Plotted Against Percentage Unbalance, Showing the Linear Relationship Predicted by Eq 7.

If the error $\Delta k/k$ is multiplied by the ratio $(q - q_0)/q = ck/q$, which is the ratio of the error heat flow in the specimens to the total error flow, only the error due to error heat flow in the specimens remains, and this is independent of k. This is apparent when both sides of Eq 6 are multiplied by $(q - q_0)/q$:

$$\frac{\Delta k}{k}\left(\frac{q - q_0}{q}\right) = \frac{L}{A}\frac{\Delta\theta}{\theta}c.$$

Therefore, the fraction of the total error, due to error heat flow in a 1-in. thick specimen is:

$$\frac{\Delta k}{k}\left(\frac{q - q_0}{q}\right)\frac{1}{L} = \frac{c}{A}\frac{\Delta\theta}{\theta} \quad \ldots\ldots (7)$$

In Fig. 11, $\Delta k/k\,(q - q_0/q)\,1/L$ has been plotted against $\Delta\theta/\theta$ for plate A for all specimens tested. The slope of the line is equal to the value of c/A for the plate.

Equation 6 might appear to indicate that the unbalance errors are inversely proportional to the test area of the plate. However, c and q_0 also depend upon A, q_0 being directly proportional to the perimeter of the test area. Since, however, the ratio of perimeter to area for a square or a circular plate decreases as area increases, the net effect is that the unbalance error decreases as plate size increases.

It has been shown that for a given heater plate the largest percentage errors due to unbalance occur with large specimen thicknesses and low conductivity materials. The thicknesses of the rock wool specimens, the test results for which are shown in Fig. 3, represent the ASTM maximum permissible thicknesses for the respective plate dimensions. Rock wool represents a typical low conductivity material. The temperature difference employed in these tests, 40 F, is the minimum permitted by ASTM. The maximum temperature unbalance allowed by ASTM ($\Delta\theta \times 100/\theta = 0.75$) results in an error in measured conductivity for these specimens of 2.34 per cent for plate A, 5.24 per cent for plate B and 6.05 per cent for plate C. Thus excessive errors due to unbalance can occur even when the ASTM requirement is met. In Britain and continental Europe still greater errors can result since smaller hot plate to cold plate temperature differences are common and since, at least one test method, that of RILEM, (1) permits a larger value of percentage unbalance ($\Delta\theta \times 100/\theta = 1.0$).

With the aid of Eq 6:

$$\frac{\Delta k}{k} = \frac{L}{A}\frac{\Delta\theta}{\theta}\,(q_0/k + c),$$

the error due to any unbalance $\Delta\theta$ when testing a specimen of conductivity k and thickness L, with a temperature difference θ and with a plate of test area A, may be calculated if the unbalance sensitivity parameters q_0 and c of that particular plate are known. Conversely if the unbalance errors are to be kept below a certain limiting value $\Delta k/k$, then the maximum unbalance $\Delta\theta$ that can be tolerated can be calculated. The best heater plate in this respect will be the one with the lowest values of q_0 and c.

One of the unbalance sensitivity constants, q_0, may be roughly determined by calculation if the detailed design and geometry at the gap are known. The total heat transfer directly across the gap for a temperature difference of 1 F, q_0, may be separated into the following components for the purposes of calculation:

1. Conduction or convection transfer across the air gaps,
2. Radiation transfer across the air gaps,
3. Conduction through the plate material which crosses the gap,
4. Conduction through thermocouple wires crossing the gap, and
5. Conduction through the two test area heater leads which cross the gap.

In making the calculations it is assumed: (*a*) that these components are all non-interacting parallel mechanisms of heat transfer; (*b*) that a 1 F unbalance measured between the test and guard surface plates represents a temperature difference of 1 F for each of the above heat transfer components; and (*c*) that corner effects may be neglected. Of great importance, then, to the value of q_0 are:

1. The over-all thickness of the heater plate,
2. The gap width,
3. The perimeter of the test area and hence the size of the plate,

4. The number and thickness of thermocouple wires crossing the gap, and
5. The emissivity of the metal face plates.

The calculated and experimental values of q_0 for the three plates are shown in Table III. The agreement between calculated and experimental values suggests that calculated values are adequate for purposes of designing heater plates but should not be relied upon for the prediction of the actual error due to heat flow directly across the gap. The calculation of the error heat flow in the specimens to give the value of c, the second unbalance sensitivity parameter, is much more difficult. The determination of the error heat flow in the specimen for a given unbalance by mathematical analysis and by use of the relaxation technique will be described in a future paper.

TABLE III.—COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES OF q_0.
(Btu per hr deg Fahr unbalance)

Heater Plate	Experimental	Calculated
A..............	0.12	0.12
B..............	0.33	0.24
C..............	1.84	1.97

Values of q_0 and c for any plate may be determined experimentally by the performance of at least two unbalance tests on at least two different conductivity materials and analyzing the data as described above. Two homogeneous materials of widely different thermal conductivities, for example, 0.25 and 2.0, should be selected for this purpose in order to achieve the maximum accuracy in the determination of these constants. For the same reason, it is suggested that the maximum permissible specimen thickness be used for these measurements.

SUMMARY AND CONCLUSIONS

The magnitude and variation of the errors in thermal conductivity due to small temperature unbalances between test and guard areas in three different guarded hot plates have been investigated. The following conclusions may be drawn.

The percentage error due to unbalance is:

(*a*) Directly proportional to the ratio of the unbalance to the temperature difference between hot and cold plates;

(*b*) Directly proportional to the thickness of the specimen tested;

(*c*) Strongly dependent upon the conductivity of the specimen tested, being greater the lower the conductivity;

(*d*) Dependent upon the size and design of the heater plate.

The relationship between the error due to unbalance and these factors can be expressed by the formula:

$$\frac{\Delta k}{k} = \frac{L}{A} \frac{\Delta \theta}{\theta} \left(\frac{q_0}{k} + c \right)$$

It has been shown how, using this relation, the maximum tolerable unbalance to achieve a certain accuracy in conductivity may be calculated provided that two heater plate constants are known. These constants, q_0 and c, are defined, for unit temperature difference between test area and guard ring, as the heat transfer directly across the gap and the error heat flow in specimens of unit conductivity, respectively.

A poorly designed heater plate, that is, one with large values for q_0 and c, will require closer balance control to achieve a given accuracy than a well-designed plate under the same test conditions. The value of q_0 will be low when: (*a*) the over-all thickness of the heater plate is small; and (*b*) the gap conductance is low. The value of c will be low when the gap width is large. However, as the gap width increases, the size of the test area becomes more indefinite and the distortion of the isothermals becomes greater. For a given gap width the effect of this distortion is smaller the larger the test area. It would therefore seem more appropriate to specify a maximum value for the ratio of gap width to length of side of test area than to specify a maximum value for the gap width.

The fact that unbalance errors as high as 6 per cent were obtained, even when the ASTM balance requirement of $\Delta\theta \times 100/\theta \leqslant 0.75$ was satisfied, indicates that much better balance control than is required by the ASTM method is necessary for accurate measurements. Instead of specifying a maximum value for $\Delta\theta \times 100/\theta$, a maximum error due to unbalance, say 0.5 per cent, should be specified. The degree of balance required to achieve this accuracy can be calculated prior to any test if the constants for the heater plate have been determined.

Acknowledgments:

The authors greatly appreciate the continued interest and encouragement of Mr. N. B. Hutcheon and wish to express their thanks to Messrs. J. G. Theriault and C. St. Jacques who carried out most of the experimental work. This paper is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director.

REFERENCES

(1) Réunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions (RILEM), *Bulletin*, No. 19, pp. 1–12 (1954).

(2) C. F. Gilbo, "Experiments with a Guarded Hot Plate Thermal Conductivity Set," Symposium on Thermal Insulating Materials, Am. Soc. Testing Mats., pp. 45–55 (1951). (Issued as separate publication *ASTM STP No. 119.*)

(3) A. J. A. Roux, S. J. Richards, and W. M. H. Rennhackkamp, "The Measurement of the Thermal Properties of Building Materials—The Thermal Conductivity of Thin 12" Square Samples," *Series DR-6*, South African Council for Scientific and Industrial Research, National Building Research Inst., Nov., 1950.

(4) A. Pascal, "La Mesure de la Conduction Thermique des Matériaux du Bâtiment," Inst. Technique du Bâtiment et des Travaux Publics, No. 90, June 1955, pp. 583–598.

(5) M. S. van Dusen, "The Thermal Conductivity of Heat Insulators," *Journal*, Am. Soc Heating and Ventilating Engrs., Vol. 26, No. 7, pp. 625–656 (1920).

(6) C. D. Niven, "On the Testing of Fiber-Board Insulation," *Canadian Journal of Research*, Vol. 7, pp. 115–130 (1932).

(7) T. M. Dauphinee and S. B. Woods, "A d-c Amplifier for Temperature Control," *Review of Scientific Instruments*, Vol. 26, No. 7, p. 693 (1955).